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ABSTRACT

There has been concern over the validity of the Algebra Diagnostic Test (ADT) used to determine the actual level of student preparation for the first quarter of calculus as taught at the University of California, Santa Barbara. It has been hypothesized that performance-based questions, along with the more traditional multiple choice questions, could provide a better assessment of students' understanding of pre-calculus materials for the Hughts-Hallet curriculum. In this study, a two-part test was written to test for ability and initiative in solving multiple level tasks and to provide for pre-calculus assessment. The performance-based test developed was a four-category examination testing algebra, function, proportion, and geometry. Qualitative and quantitative analyses were conducted to find relationships between test scores and subsequent grades in algebra, calculus for the hard sciences, calculus for the social sciences, and chemistry. The performance-based test had significant correlations with grades in all four classes. Multiple choice testing had a higher correlation than performance-based testing, but a combination of both provided the best test to predict mathematics or chemistry achievement. Mathematics and chemistry achievement are correlated with the following examination portions: symbolic manipulation of algebraic expressions, graphing of functions, symbolic properties of functions, trigonometric functions, logarithmic functions, word problems, geometry, and numerical data or number sense. Symbolic manipulation skills are statistically significant in predicting grades in all four classes. Students who display fluency with function concepts are statistically shown to be successful in calculus and chemistry classes. Six appendixes contain the developed instrument, a multiple-choice test, data scatter plots, tables of data, course descriptions, and a precalculus diagnostic test. (Contains 18 tables, 20 scatter plots, and 4 references.) (SLD)

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ED 409 369

UNIVERSITY OF CALIFORNIA
Santa Barbara

The Validity of Pre-Calculus Multiple Choice and Performance-Based Testing as a Predictor of Undergraduate Mathematics and Chemistry Achievement

A Thesis submitted in partial satisfaction
of the requirements for the degree of

Master of Arts

in

Mathematics

by

Gwen Laura Fisher

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Professor Jeffrey Stopple

December 1996

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CORRECTION

Dear Reader,

On Page 4, elasticity is defined to be

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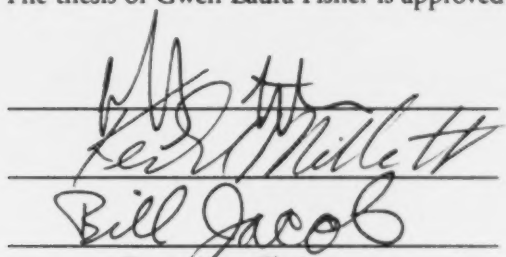
The thesis of Gwen Laura Fisher is approved



Committee Chairperson

December 1996

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I would also like to acknowledge the help of my family: my sister Ruth for her logical and statistical skills; my brother Arthur for his computer graphics wizardry; and my parents Judy and Norman, who always encouraged me to turn my dreams into reality.

Abstract

The Validity of Pre-Calculus Multiple Choice and Performance-Based Testing as a Predictor of Undergraduate Mathematics and Chemistry Achievement

by

Gwen Laura Fisher

There has long been concern over the validity of the Algebra Diagnostic Test (ADT) to determine the actual level of student preparation for the first quarter of calculus at the University of California at Santa Barbara. For an understanding of the Hughes-Hallet calculus text from Harvard University, it is believed that students need more problem solving abilities as well as the initiative to do embedded tasks within a given problem. It is hypothesized that "performance-based" questions, along with the more traditional multiple choice questions, could provide a better assessment of students' understanding of pre-calculus materials for the Hughes-Hallet curriculum. In this study, a two-part test was written with the goal to test for ability and initiative in solving multiple leveled tasks, as well as traditional pre-calculus assessment. The Performance-Based Test analyzed is a written exam consisting of four categories: Algebra, Function, Proportion, and Geometry. A thirty question, shortened version of the original ADT was created to compare to the performance-based test. Qualitative and quantitative analysis were conducted to find relationships between the students performance on the exams and the subsequent grades in the students' first quarter of algebra, calculus for the hard sciences, calculus for social sciences, and chemistry. The performance-based testing we tried has a significant correlation with grades in all four classes analyzed. In every case, multiple choice testing had a higher correlation than the performance-based testing, but a combination of both provided the best test to predict mathematics or chemistry achievement. Mathematics and chemistry achievement are correlated with the following portions of the exam: symbolic manipulations of algebraic expressions, graphing of functions, symbolic properties of functions, trigonometric functions, logarithmic functions, word problems, geometry, and numerical data or number sense. Symbolic manipulation skills are statistically significant in predicting grades in all four classes. Students who display fluency with function concepts are statistically shown to be successful in calculus and chemistry classes.

Contents

Acknowledgments iv
Abstract v
Introduction 1
The Performance-Based ADT 1
The Multiple Choice ADT Exam 2
Overall Scores 2
Errors in Grading 3
Method of Quantitative Analysis 4
Calculus 3A Quantitative Analysis 5
Calculus 34A Quantitative Analysis 7
Mathematics 15 Quantitative Analysis 8
Chemistry 1A Quantitative Analysis 8
Calculus 3A Qualitative Performance-Based Analysis 9
Conclusion 12
Topics for Further Analysis 14
References 15
Appendix 1: Courses 17
Appendix 2: The Performance-Based ADT 19
Appendix 3: The Multiple Choice ADT 25
Appendix 4: Data Scatter Plots 37
Appendix 5: Tables 59
Appendix 6: Precalculus Diagnostic Test 79

The idea of this thesis began with an informal conversation about math education with my advisor Professor Bill Jacob. We discussed different questions that were yet unanswered in the realm of math education, and it was at our second meeting that the question of pre-calculus assessment arose. Apparently, there had long been concern over the validity of the Algebra Diagnostic Test[1] (ADT) to determine the actual level of student preparation for the first quarter of calculus at the University of California at Santa Barbara (UCSB). Professor Jacob believes the ADT was of little or no significant use in the determination of success in the new calculus curriculum as taught in Calculus 3A.[2]

The ADT was introduced to test preparation for a traditional college calculus curriculum. However, within the past few years both of the calculus courses at UCSB have been changed to newer, more applied curriculums. For an understanding of the Hughes-Hallet text from Harvard University (Copyright 1994, by John Wiley & Sons, Inc.) used in Calculus 3A, it is believed that students need more problem-solving abilities as well as the initiative to do embedded tasks within a given problem. It is hypothesized that the ADT's multiple choice format tests these abilities inadequately, instead emphasizing more rote procedural and notational abilities. This hypothesis is supported by De Lang in "Assessment: No Change without Problems" (1995).

With the initiation of a new approach to teaching college calculus, there comes the need for new modes of pre-calculus assessment (De Lang, 1995; National Council of Teachers of Mathematics [NCTM], 1995). It was Professor Jacob's belief that "performance-based" questions, along with the more traditional multiple choice questions, could provide a better assessment of students' understanding of pre-calculus materials for the Hughes-Hallet curriculum. Indeed, the literature shows that there needs to be a shift away from using solely multiple choice exams to using more varied approaches to assessment (Baxter, Shavelson, Herman, Brown, & Valdez, 1993; De Lang, 1995; NCTM, 1995). In this study, a two-part test was written with the goal to test for ability and initiative in solving multiple leveled tasks, as well as traditional pre-calculus assessment.

The Performance-Based ADT

The Performance-Based Test is a written exam consisting of four questions: Algebra (8), Function (10), Proportion (6), and Geometry (6) with points weighing as indicated. Utilizing Professor Jacob's numerous years of academic experience, we created a weighting scheme. The weighting scheme was only based on Professor Jacob's presupposition, because one objective of this study is to determine relative significance

[1] CSU/UC Mathematics Diagnostic Testing Project Publishers, 1993.

[2] There is an official description of all courses discussed in this paper in the Appendix 1.

of the text portions in predicting calculus achievement. Thus, the only reason to give point values is to have a numerical score to report to the Summer Orientation Department to be used for student advisement. The test and its rubric were written with the idea of testing for various levels of understanding and initiative to answer the question being asked. A draft version of the test was administered to approximately one hundred Mathematics 15 students for the purpose of viewing a sample of responses. Minor changes were made in both the test and the rubric with regard to this sample.[3]

The Multiple Choice ADT Exam

Professor Jacob and I created a shortened version of the old ADT. First, we independently classified each of the sixty questions into one or more of the categories identified below. We then created a thirty question exam maintaining the partition but reducing the number of questions in each category. The first number after each category indicates the number of such problems in the old, sixty question exam. The second number indicates the number of questions in the revised, thirty question exam. Because some problems are determined to fall into more than one category the total of the first (second) numbers is more than sixty (thirty).

1. Symbolic Manipulation of Algebraic Expressions (including value substitution, roots, inequalities, etc.), 30, 14.
2. Functions: subclassify
    a. General Graphing, 9, 8.
    b. Symbolic Properties, 5, 3.
    c. Trigonometry, 8, 3.
    d. Logarithms, 5, 3.
3. Word Problems, 8, 3.
4. Geometry, 3, 2.
5. Numerical Data or Number Sense, 5, 2.

The point to reducing the old ADT to half of its original length was to allow time for the students to complete both multiple choice and written portions of the exam in the allotted time of one and a half hours, divided into forty-five minutes for each of the two parts.[4]

Overall Scores

A total of 1733 students' scores were recorded. The mean of the entire exam is 27.1 out of 60, with a standard deviation 10.7. There were 42.2% of the students who passed with a score of 30 or better with recommended placement into Calculus

[3] The questions and their accompanying rubrics are featured in Appendix 2.
[4] The thirty point version of the Multiple Choice ADT is featured in Appendix 3.

34A, and 22.3% passed with a 36 or better with recommended placement into Calculus 3A or 34A depending upon their major.[5] Calculus 34A or Calculus 3A is required for social science majors, and Calculus 3A is required for mathematics and science majors.

The mean score on the Multiple Choice ADT is 14.7 out of 30 with a standard deviation of 5.7, where each question is worth one point. There were 49.7% of the students who passed this portion of the exam with a score of 15 or better to be recommended to take Calculus 34A, and 31% passed with a score of 18 or better to be recommended to take Calculus 3A or 34A. On the Performance-Based ADT, there are four problems of varying points, as described in Appendix 2. On this portion, the students scored a mean of 12.3 out of 30, and standard deviation 6.5, where 36.6% passed to be recommended to take Calculus 34A, and 22.3% to be recommended to take Calculus 3A or 34A. Note that a student need not pass both portions independently. It is the combined score that is used, which here after will be referred to as the Final Recorded Score.

Correlations between the two portions of the exam were calculated, yielding a correlation of 0.587. The correlation is statistically significant at the one percent level, indicating that students who did well on one portion tended to do well on the other portion also. See Table 1 for a summary of these results.[6]

Errors in Grading

There were 1745 exams. I graded approximately half (900) of the Performance-Based ADT exams, Fernando Olarte graded 700, and Professor Bill Jacob graded 200. To compare the reliability of multiple graders utilizing the same rubric, I graded a random sample of the other graders exams a second time. The sample sizes were 50 (6%) and 20 (10%) for Olarte and Jacob, respectively.

Percent error is measured by taking a sum of the absolute difference of the scoring of each of the four exam questions and then dividing by the total number of possible points (30 points per exam). Accuracy is then one minus the percent error. The results of the cross samples with Olarte and Jacob achieved an overall accuracy of 94.0% and 96.5%, respectively.

The thirty point multiple choice portion of the exam was graded by a scantron reader. In the original recordings of the scores, only the final grade was saved. So, a total of 1745 scantrons were reread through the same scantron reader to obtain results to individual exam questions. Of these, only 410 (23.5%) recorded the same final score both times, 812 (46.5%) had a one point discrepancy, 496 (28.4%) had a two point discrepancy, and 28 exams (1.6%) had a discrepancy of 3 to 16 points

[5] The Department of Mathematics does not enforce placement, but the scores are reported to the academic advisor.

[6] All tables are featured in Appenix 5

between the two readings. The sum of the absolute differences between the two sets of scores is 1990 points, and the standard deviation is 1.41. The percent error is 3.8%, which gives an accuracy of 96.2%, roughly the same as the performance-based exam.

I believe that the reason for the inconsistency between the two readings was predominantly caused by students improperly marking their scantron bubbles. When the bubbles are not fully and heavily shaded, the scantron reader is likely to pick up the mark only on some occasions.

Method of Quantitative Analysis

The data collected on each of 1733 students consists of thirty answers to the Multiple Choice exam, four Performance-Based solutions, the Final Recorded Score, and the letter grades received in any of Calculus 3A, Calculus 34A, Mathematics 15, and Chemistry 1A from the Fall quarter of 1995. The letter grades were converted to a grade point ranging from zero for an F to 4.3 for an A+, in increments of a third of a grade.

The first analysis consists of viewing scatter plot graphs where class grade points are on one axis, and test scores are on the other. For each of the four classes, there are five independent test scores considered, namely Multiple Choice, and each of the four Performance-Based questions: Algebra, Function, Proportion, and Geometry.[7] For each of the four classes discussed in this paper, a series of ordinary linear regressions were run, and in each case the class grade is the dependent variable. The independent variables consist of different combinations and partitions of the ADT scores. The values reported are as follows for each linear regression $y=\alpha+\boldsymbol{ß}\bullet\mathbf{x}$, where $\boldsymbol{ß} = (ß_1,\ldots, ß_i,\ldots, ß_n)$ and $\mathbf{x} = (x_1,\ldots, x_i,\ldots, x_n)$ are vectors:

y	=	the dependent variable, typically the course grade
x_i	=	the independent variables, typically the test scores
$ß_i$	=	the regression coefficient of x_i
ε_i	=	elasticity
	=	$(dy/y)/(dx_i/x_i)$
	=	$ß_i$ [(average of y)/(average of x_i)]
t-statistic	=	gives the significance of $ß_i$
Adjusted R^2	=	gives percent of the variance of y for which the regression accounts

In some cases, I chose to do a more substantial quantitative analysis where there seemed to be more involved than an ordinary linear regression will show.

[7] Scatter plots are featured in Appendix 4, sectioned by class.

Calculus 3A Quantitative Analysis

A total of 517 students who took the ADT subsequently took Calculus 3A for a letter grade in the fall of 1995. The students achieved a mean score of 35.4 out of 60 with a standard deviation of 8.5. On the Multiple Choice ADT, the students achieved a mean score of 18.7 out of 30 with a standard deviation 4.8. On the Performance-Based ADT, the students achieved a mean score of 16.6 out of 30 with a standard deviation 5.4.

I found that each of the two portions of the ADT is statistically significant in determining the grade points of Calculus 3A students; however, the Final Recorded Score only accounts for 11.4% of the variance. The Multiple Choice ADT alone accounts for 11.0% of the variance, and if we reweight the two exams so that Multiple Choice is three-fourths and Performance-Based is one-fourth of the final score, we can then account for another 1.4% of the variance in the grade points of Calculus 3A students. (See table 2.) The reader should be reminded that the weighting scheme of the Performance-Based ADT was arbitrarily chosen at the onset of the study for reporting scores.

What we then see from these numbers is that the utility of the Multiple Choice ADT portion is slightly increased by the addition of the Performance-Based ADT, but that the use of this performance-based exam alone is minimal for the prediction of Calculus 3A grades.

A linear regression was run between each of the four parts of the Performance-Based ADT and Calculus 3A grades. Within the Performance-Based ADT we see that Function and Proportion are both statistically significant in determining Calculus 3A grades with best relative weighting at approximately 4:3, respectively. Neither Algebra nor Geometry is found to be statistically significant. (See table 3.) These results are confirmed by the scatter plots.

Only two out of the fifty-three Calculus 3A students who received a perfect score on the Function problem did not receive a high enough grade to continue calculus[8], and less than 10% who did receive at least a three on the Function problem did not receive a high enough grade to continue (N=112). In other words, those who scored at least a three on Function tended to pass Calculus 3A. However, of those who received no credit on this problem, 21% received an A and a total of 79% earned a C or better. What this tells us is that the people who could display a basic ability to graph this function are adequately prepared to pass calculus. Yet for those who did not display a general understanding of the Function problem, we can make no such conclusions for passing or failing in Calculus 3A.

[8] The Mathematics Department enforces a grade of C or better to continue.

Three linear regressions were run considering the relationship between Calculus 3A grades and the Multiple Choice ADT. We first considered the total score, which accounted for 11.0% of the variance. Then, to see which types of questions are of importance, the Multiple Choice exam was partitioned with respect to the partition stated earlier, i.e. Functions, Numerical Data, Symbolic Manipulations, and Geometry. We find that Functions, Symbolic Manipulations, and Geometry are statistically significant in their relationship to Calculus 3A grades, with respective ratios of approximately 2:3:1. (See table 4.) The Functions section was then divided into the four categories of Graphing, Symbolic Properties, Trigonometry, and Logarithms. With the rest of the four parts of the partition unchanged, a linear regression was run among these eight variables and Calculus 3A grades as the dependent variable. Symbolic Manipulations and Geometry remain statistically significant, and we find within the category of Functions that Symbolic Properties is the only statistically significant category. (See table 5.)

When we compare the adjusted R^2 values of each of these three regressions, there is less than a one percent absolute difference among them. Hence, the gain achieved from this analysis is to show which topics have utility in preparing students for Calculus 3A. In summary, a good understanding of proportional reasoning, functions (especially symbolic properties of functions), symbolic manipulations of algebraic symbols, and geometry is important in determining success in Calculus 3A. On the other hand, an ability to display an understanding of the relationship between algebraic representations of functions and their graphs, trigonometry, logarithms and word problems is not shown to have statistical significance in determining Calculus 3A grades. Although the two Numerical Data questions are not shown to have statistical significance, the t-statistic and elasticity value are high enough that I hypothesize that a larger sample of this type of question would show this category to be significant. In support of this hypothesis is that the Proportion question of the Performance-Based exam was found to be significant and does in fact test for an ability to work with numerical data.

Correlations were calculated between the grade point of 517 Calculus 3A students and each of the thirty multiple choice questions. Problem 18 has the highest correlation of 0.225 ($p<0.001$). A possible conclusion that one might draw from the analysis of correlations is that a fundamental skill for succeeding in Calculus 3A is factoring out of a radical sign (question 18). However, this is not a skill that is frequently used in 3A if it is used at all. It is my belief that this is really showing us that a student's general mathematical preparation and willingness to learn notational manipulation is important. Much of the basis of calculus 3A is the concept of the function, and this exam supports the notion that a clear understanding of this concept will help a student succeed in 3A. Many different properties of functions seem to be relevant, especially understanding how to substitute a value of x into a function f (x) (questions

4 and 30). Students need to be able to interpret the graph of a function in terms of the data points (question 9). Interpretations of the definition are also useful (questions 9 and 19). The sine (question 22), absolute value (question 28), and linear (question 10) functions show relevance. Conversely, algebraic manipulation of absolute value symbols (question 1), as well as the graphs of log (question 23), exponential (question 29), and parabolic (question 27) functions appear to be irrelevant. However, it seems to be that fluency in algebraic manipulation of linear functions, logarithms, quadratics, and numbers with real valued exponents are needed skills to succeed in Calculus 3A.

We now turn to the Pass/Fail chart of Calculus 3A students (table 6). Recall that one needs a score of 36 or better to be recommended for Calculus 3A. Almost half, 45.6% of the students who passed Calculus 3A scored below 36 points. In addition, 29.4% of the students who failed Calculus 3A, with a C minus or less, received a score of 36 points or more. This chart brings into question what the lowest passing grade should be. It can be determined by the department administering the exam which would be the most appropriate least score for a passing score on the ADT.

Calculus 34A Quantitative Analysis

When we compare the linear regression results of ADT scores and subsequent Calculus 34A grades with those of Calculus 3A, we find that Calculus 34A has a less significant relationship to exam scores overall. The Multiple Choice ADT accounts for only 6.4% of the variance when regressed as the only independent variable against Calculus 34A grades. If we choose Multiple Choice and Performance-Based as independent variables with Calculus 34A grades as the dependent variable, we can account for 9.3% of the variance. Hence, performance-based assessment when combined with multiple choice does have utility in predicting Calculus 34A grades, using an optimum ratio for point weighting of 3:4, respectively. (See table 7.)

If we consider each of the four parts of the Performance-Based ADT as independent variables and Calculus 34A as the dependent variable in a linear regression, we see that the Function problem is the only significant independent variable. Also, the elasticity of Calculus 34A and Function is only about a third of the elasticity of Calculus 3A and Function. Therefore, this problem has a greater utility in predicting Calculus 3A rather than Calculus 34A achievement. (See table 8.)

The previously discussed Multiple Choice ADT partitions were then used for two more regressions with Calculus 34A as the dependent variable. Independent variables are shown in tables 9 and 10. Both regressions show Symbolic Manipulations to be the only significant independent variable. I believe that the results of Performance-Based Algebra question may be of some utility because the Algebra solution requires a good understanding of Symbolic Manipulation skills. In particular, to solve the Algebra problem, one needs to know how to set up algebraic expressions for area and

perimeter, to substitute a variable, and to find the roots of a binomial equation Although Algebra was not found to be statistically significant, the t-statistic is high enough at 1.894 that I hypothesize the ability to solve this problem is an important for Calculus 34A achievement.

Mathematics 15 Quantitative Analysis

Elasticity of the test questions is lowest overall with Mathematics 15, relative to the four classes analyzed. However, the Performance-Based ADT is shown to be a relatively significant contributing factor in determining Mathematics 15 preparation. Table 10 shows that the Performance-Based ADT, as constructed, should comprise roughly a third of the total points of the exam.

The only one of the four questions of the Performance-Based ADT shown to be relevant in predicting Mathematics 15 achievement is Proportion. Function and Geometry are not shown to be significant. (See table 12.) Although Algebra is also not shown to be statistically significant at the 5% level, with a t-statistic of nearly 2, I hypothesize that the skills needed to solve the Algebra question may in fact be important for Mathematics 15 preparation. This hypothesis is supported by the fact that the category of Symbolic Manipulation of the Multiple Choice ADT is statistically significant and the Performance-Based Algebra question involves a substantial amount of symbolic manipulation skills to solve successfully. In fact, when the Multiple Choice ADT is partitioned with respect to the previously discussed partitions, Symbolic Manipulation is the only category to show statistical significance as a predictor of Mathematics 15 grades. See tables 13 and 14 for a summary of these results.

The fact the Function problem is insignificant can at least be partially explained by the fact that only 9 of 256 students passed the Function problem with 3, 4, or 5, and of the five who received a perfect score, only one passed Mathematics 15 with a high enough grade to continue to take calculus. If these students are compared with the sample of 53 Calculus 3A students, the five Mathematics 15 students should have already been prepared to take calculus with less than a four percent chance of failing. Clearly, more is happening in the lives of these five students than this test is able to show.

Chemistry 1A Quantitative Analysis

Surprisingly, the ADT is shown to account for a higher amount of variance in Chemistry 1A grades than for any of the three mathematics courses analyzed. Again it is found that this Performance-Based ADT is of less value than the Multiple Choice ADT is; however the Performance-Based ADT is a useful complement to the Multiple Choice. An optimum ratio for test weighting would be about 4:1, Multiple Choice to Performance-Based. See table 15 for a summary of these results.

The Function problem of the Performance-Based ADT is again found to be of

high utility; however Algebra was in fact found to have a higher elasticity, with both questions significant at the one percent level. Unlike what the Calculus 3A regressions showed, Geometry was a significant indicator of Chemistry 1A achievement, and Proportion was not found to be significant. See table 16. These results are reiterated in the analysis of the Multiple Choice ADT where Functions, Symbolic Manipulations, and Geometry are all found to be statistically significant. (See table 17.) When the Functions category is further partitioned, it is shown that the Trigonometry section accounts for most of the utility of Functions in predicting Chemistry 1A achievement. (See table 18.)

Calculus 3A Qualitative Performance-Based Analysis

A random sample of 150 of the 527 exams of Calculus 3A students were chosen for a qualitative analysis of the students' performance-based responses. The mean grade points among the whole sample and the subsample of 150 differ by less than 0.03 of a grade point, with mean of the subsample being 2.65. The standard deviations differ by less than 0.02 of a grade point. The two samples have the same median of 3.0 and mode of 4.0. Thirty-two (21.3%) of the sample of 150 received a grade of C minus or less.

The qualitative analysis consisted of recording the different types of student responses to the four Performance-Based exam questions. Correct, nonstandard and incorrect responses were recorded with regard to the Calculus 3A grade received by the student examinee.

The most striking results are found in the responses to the Function question. Fourteen of the students created a data table of time and distance to aid them in the creation of their graphs. All were able to produce a linear graph, and every one of these students passed Calculus 3A with mean grade points being 3.36. Nine students generated an algebraic expression to represent their graph.[9] All of these nine passed Calculus 3A with mean grade points being 3.44. Hence, the ability and initiative to turn word problems into charts and algebraic expressions is shown to have an impact on students' Calculus 3A achievement.

In the sample of 150, there are thirty-one students who received a zero on the function question, indicating no linear graph of distance versus time was drawn. However, seventeen of these students drew some sort of graph indicating that Fresno is between Bakersfield and Sacramento, many of which indicate the appropriate distances and times between the cities. Although the seventeen students' mean grade points differs from the sample of 150 by less than a tenth of a grade point, there does exist a difference between these seventeen and the rest of the students who received no credit on the Function problem. The seventeen who created the alternate graph have

[9] Expressions were of one of the following general forms: d=65t, d=104-65t, or d=|104-65t|

mean grade points of 2.71, while the remaining fourteen have mean grade points of 1.98. This two-thirds of a grade difference between the two groups may indicate that students who take an initiative to map a given problem in some tangible form, even if they are not able to produce a correct answer, tend to do better in Calculus 3A than the students who do not.

In the Proportion question, data is given in the number of daylight hours, and the answer is to be given in how many hours later the sun sets. Hence, the final step in solving this problem is to divide daylight hours by two. There were eleven students who realized the need for this step. The mean grade points of this group is 3.06, over a third of a grade higher than the sample mean.

The most common obstacle of the Proportion question is in dealing with units, and there appear to be two basic forms in the manifestation of this obstacle. The first is demonstrated in those students who seem to ignore the fact that the data is given in hours, and their answers are written in minutes, i.e. 0.6 minutes, 6 minutes or 360 minutes. Eleven students wrote one of these answers and their mean grade points is 2.25 or roughly a third of a grade lower than the sample mean. The other obstacle is demonstrated by the sixteen students who attempted to convert hours to minutes unsuccessfully. Five of these sixteen received a C minus or less in Calculus 3A, and the mean grade points is 1.96, almost two thirds of a grade lower than the sample mean. Hence, the ability to successfully deal with units and conversion of units is shown to be an important skill for Calculus 3A students.

Within the Geometry question, there is a standard way to solve for each of the unknowns x and y. Most students who solved for x correctly used the Pythagorean Theorem, and exactly ten percent of the sample merely cited the special case where isosceles right triangles have sides in a $1:1:\sqrt{2}$ ratio. There was no significant difference shown between these two groups. Four students used a fallacious version of the Pythagorean Theorem, but all passed Calculus 3A with a C or better. The two students who stated x to be the circumference of the circle have mean grade points of only 1.80, substantially lower than the sample mean. Two students assumed x to be a side of an equilateral triangle, and unexpectedly both of these students received an A in Calculus 3A. There are three clever students who used trigonometric properties of triangles to solve for x, where two used sine, and one used cosine. All three of these students passed Calculus 3A with mean grade points comparable to the sample mean.

The simplest way to solve for y is to see that y is one diagonal of a rectangle where the other diagonal is the radius. There was one thoughtful A$^+$ student who remarked:

> y is the diagonal. (important?)
> Why is the diagonal important?
> Why, is the diagonal important?
> y is the important diagonal.

Another insightful student actually considered the limiting case of y, where the rectangle degenerates to a line. This student received an A in Calculus 3A, as is to be expected.

The types of errors made by the students on this Geometry problem are numerous. The most common error is to assume that y is a side of some specific type of triangle, which it is not, yet for the simplicity of drawing the diagram on a computer, a 3:4:5 right triangle was used. Seven of the 150 students recognized this resemblance, and their mean grade points is comparable to the rest of the sample. Fifteen other students incorrectly assumed y to be a side of a 30°-60°-90° or 45°-45°-90° or some other special triangle, and this group's mean grade points of 2.31 is about a third of a grade lower than the sample mean. Another common error is to assume that x and y are in some way proportional to one another. Fifteen students displayed this misconception, but their mean grade points is comparable to the sample mean.

Clearly, finding the length of the diagonal y is an insight problem. The proof of the solution is only two lines; thus one either sees it or does not. There is really no computation necessary. This question, in fact, duped a number of mathematics graduate students in an informal quandary, and even a professor or two was temporarily stumped by the subtlety of the solution. The results of this analysis show that having quick geometric insight will not make or break a Calculus 3A student. Aside from the few students who used innovative techniques to solve the Geometry problem, there is no assumption to be made about the relationship between the quality of a student's response and his subsequent Calculus 3A achievement.

On the Algebra problem, the standard solution of the algebraic equation is to factor it into linear factors. The two other, less common solutions are to use the Quadratic Equation and a simple "guess and check" method. For any student who managed to find the correct numbers, but showed no justification for his answer, I assume the student used the "guess and check" method. Twenty-four students guessed correctly, and their mean grade points is comparable to the sample mean. Seventeen students correctly substituted into the Quadratic Equation, and again, this group resembles the sample of 150. The two students who used an incorrect form of the Quadratic Equation both received an A in Calculus 3A. The one clever student who attempted successfully to "complete the square" barely passed Calculus 3A with a C. Seven students made errors in algebraic manipulations of symbols, most while trying to use the Distributive Property, and all but one passed with a C or better.

There were only six students who attempted the problem but failed to set up the equation properly. Of these six, two failed Calculus 3A with a C minus or less, and both made the same error by writing an equation of the form $(x+50)y=2400$. They used two variables in an equation where there should only be one. This is the only qualitative response on the Algebra question that I found to have any relevance to Calculus 3A achievement.

Conclusion

My analysis of thirty-four questions given to 1,733 students has led me to the following conclusions:

- Some exam topics have statistically significant correlations with UCSB's mathematics and chemistry grades
- The attempt of students to solve performance-based problems has fundamental value for calculus students
- There are mathematics topics of that do not correlate with UCSB's students' mathematics and chemistry achievement
- Performance-based testing is a significant predictor of grades for a large sample
- Our exam is a relatively insignificant indicator of how an individual will be graded in subsequent mathematics courses

The fundamental utility of this study lies in what it tells us about our university freshmen who are able to succeed in UCSB's mathematics and chemistry courses. This study indicates that fluency in symbolic manipulations and mathematical notation are important for success in the college courses studied.

In all eight of the Multiple Choice ADT regressions, symbolic manipulation skills are statistically significant in predicting grades in the first quarter subsequent to taking the exam. This indicates that an ability to manipulate algebraic notation is necessary for success with the curriculum in the texts. (Often, appropriate notation greatly facilitates application of concepts. Did you ever try to use Roman Numerals for long division?) The Harvard text uses manipulations in many of the examples and requires students to do algebraic manipulations to solve problems. For example, summation notation is used when defining and using the definite integral. Therefore, students cannot be expected to understand the heuristic discussion in the Hughes-Hallett calculus curriculum if they are lost in the midst of the algebraic notation. Hence, fluency with algebraic notation should be emphasized in the curriculum of algebra and pre-calculus classes.

Proficiency in function concepts will help calculus and chemistry students succeed. In particular, students who demonstrate an understanding of symbolic properties of functions tend to achieve higher grades in Calculus 3A than students who do not. Chemistry 1A students appear to benefit from knowledge of trigonometric functions. However, fluency with functions is not a significant indicator of success in Mathematics 15. The curriculum of the course includes teaching these definitions and examples of functions. In other words, one does not need to be an expert with functions before enrolling in a course on functions in order to pass the course.

Comprehending basic geometry is relevant to predicting calculus grades and is

particularly important for Chemistry 1A students. Proportional reasoning is useful for Calculus 3A and Mathematics 15 students, presumably because the concepts of slope and linear functions are prevalent in both of these courses.

Responding to performance-based questions, students who do not produce a correct answer but do display a reasonable effort have greater success in Calculus 3A than students who make no attempt. Therefore, the attempt of students to solve performance-based problems has fundamental value for calculus students. For example, students who used an invalid form of the Pythagorean Theorem or the Quadratic Equation performed better in calculus than students who made no response. Also, students who made a chart of data before plotting points of a graph have a higher than average success rate in calculus even though many graphed an incorrect solution.

My analysis has indicated the following topics not to be of value: Word problems, in general, have not clearly indicated a relationship to grade points in any class analyzed even though the mathematics students are expected to use English on homework and exams. However, no analysis was performed to determine if a significant difference exists between a math problem written as a word problem and the same problem written in symbolic form. Additionally, logarithms and graphs of standard functions appear to be insignificant in their relationships to all four classes analyzed, and trigonometry is only significant for chemistry students. Ironically, an ability to process numerical data has shown no relationship to Calculus 34A or chemistry achievement.

The performance-based test used was a significant predictor of grades in all four classes analyzed. In every case, multiple choice testing had a higher correlation than the performance-based testing, but a combination of both provided the best test for predicting course grades. Depending on the course, the optimal weight of the exam portions ranges from 1.5 to 5 multiple choice points to every one performance-based point.

Overall, The ADT is a relatively insignificant indicator of how an individual will be graded in subsequent mathematics courses even though the ADT has proven to be statistically significant in determining mathematics and chemistry achievement for a large sample. For example, consider the 61 students who took Calculus 3A and scored 25 or lower on the ADT. Table 6 shows that 47.5 percent of these students passed Calculus 3A with a C or better. If the passing score is 31, then 125 Calculus 3A students scored lower than this with 61.6 percent passing Calculus 3A with a C or better. The official passing score is 36, and here 293 of the 553 students scored lower than this, and 68.9 percent of this 293 passed Calculus 3A with a C or better. Clearly, no student should be denied the opportunity to enroll in a course based on his ADT score. These numbers help to explain why the ADT was shown to never account for any more than 17% of the variance of grades in any of the four classes analyzed. Clearly, no student should be denied the opportunity to enroll in a course based on

his ADT score; this is an important policy decision.

The ADT does have value as an advisory exam for class placement. In every class analyzed, the relationship between the ADT and course grades is statistically significant at the 0.1% level. Therefore, for any large sample of students, the ADT is a useful predictor of the group will achievement in subsequent courses.

Topics for Further Analysis

This study provides data which can be analyzed for further research. A qualitative analysis of the Performance-Based ADT can be conducted to find relationships to Calculus 34A, Mathematics 15 or Chemistry 1A grades. To devise a better assessment exam to be used for class placement, course grades can be compared to specific multiple choice question responses and groups of questions. In addition, the results can be compared with either high school grades or SAT scores to determine which of the three is most accurate. Finally, one may be able to use this data to find where our university freshmen are most deficient in mathematics skills.

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Appendix 1

Courses

Chemistry 1A. General Chemistry

Chemistry 1 develops a quantitative understanding of matter providing a basis of all field science and also for understanding industrial technology and environmental problems. This course is required for majors in chemistry and most other fields of science and engineering , but can also provide a general education in science for the liberal arts student.

A. Stoichiometry, gas laws and kinetic theory, atomic and molecular structure, the periodic table, nature of solids, liquids, and solutions, phase chemical equilibria.

Mathematics 3A. Calculus with Applications, First Course

Differential Calculus including analytic geometry, functions and limits, derivatives, techniques and applications of differentiation, logarithmic and trigonometric functions.[10]

Mathematics 15. Precalculus

A function approach integrating algebra and trigonometry. Topics include: one-to-one and onto functions; inverse functions; properties and graphs of polynomial, rational, exponential, and logarithmic functions; properties and graphs of inverse trigonometric identities; trigonometric equations.

Mathematics 34A. Calculus for Social and Life Sciences

Functions from the numerical, graphical, and symbolic points of view with trigonometry in this context. Introduction to differential and integral calculus with applications to modeling in the biological sciences.

[10] The catalog description of Calculus 3A is not accurate; reader should consult the Harvard Text.

Appendix 2

The Performance-Based ADT

Algebra

Problem 1. A rectangular field has an area of 2,400 square yards. Its length is 50 yards longer than its width.

(a) Find an algebraic equation that describes this situation.

(b) What is the perimeter of the field?

This problem has a five level Rubric

The primary assessment objectives are to determine if students can convert a statement into an algebraic equation and process that equation in a standard way. Students must take responsibility to label their own variables.

0. The student cannot produce W(W+50) in any form that demonstrates understanding of the expression.
1. The student obtains an area formula W(W+50)=2400 or W(W+50)=AREA or W^2+50W=2400 or W^2+50W=AREA, but does not make progress processing this information. The student may instead produce the system of equations L*W=2400 and L=50+W.
2. In addition to 1., the student displays a correct perimeter formula, although neither the width nor the length is determined.
3. The student determines that W=30 or 80. Perimeter is not found.
4. In addition to 3., the perimeter of 220 is found. Perimeter process must be indicated either by displaying a calculation or by a picture. Minor arithmetic error is acceptable.

Function

Problem 2. Driving north on Highway 99 it is 277 miles from Bakersfield to Sacramento. Leaving Bakersfield at 8:00 a.m., you drive to Sacramento non stop at the legal speed of 65 m.p.h.. While you drive you are listening to a Fresno radio station. Fresno is 104 miles north of Bakersfield.

Draw a graph showing your **distance** from *Fresno* as a function of **time** during your drive. Be sure to label the axes on your graph and explain how 65 m.p.h. is used in making your graph.

This problem has a six level Rubric

The primary assessment objective is to determine if students understand, in a real sense, what a function is and how to graph it. Many students do not read the question carefully and simply graph distance from either Sacramento or Bakersfield. So in part, the assessment reflects whether they can (or are willing to) work carefully with the information provided. The rubric also measures communication skills for the highest score.

0. The student does not indicate a graph of a function of distance with respect to time. Note that we consider a function to have at least three points.
1. The student shows a linear increase or decrease on some region. No correct reference to 65 m.p.h. is given.
2. The student shows a linear increase on some region and labels axes properly and explains how 65 m.p.h. is used in making the graph. We accept such statements as, "65 is the slope," or, "Constant rate gives a straight line," or, "65 miles driven each hour." However, "65 is a constant rate," is not acceptable.
3. Same as 2., but with a linear *decrease* indicated.
4. The student indicates a V-shaped graph, but does not achieve all of the requirements of 5.. Credit is also given for unconventional function graphs which indicate the understanding of driving through Fresno, such a a linear function which crosses the X-axis.
5. The student indicates a V-shaped graph and labels axes properly and explains how 65 m.p.h. is used in making the graph (see 3.). Time estimates must be reasonable.

Proportion

Problem 3. The number of daylight hours in Madrid, Spain during the month of March is shown in the following chart.

day (in March)	1	11	21	31
daylight (in hours)	11.2	11.6	12.0	12.4

Estimate how many minutes later the sun sets in Madrid on April 15 than it did on March 31. Justify your assertion.

This problem has a three level Rubric

The primary assessment objectives are to determine if students recognize linear rates of change and can work with familiar units, converting them when needed.

0. No ratio, proportion, nor linearity is indicated.
1. The student demonstrates understanding that the increase in hours per 10 (or 11) days is constant or recognizes an arithmetic sequence. Either .4 (hours) or 24 (minutes) is acceptable; the units may be wrong to still receive credit.
2. The student recognizes rate of increase of .4 hours (or 24 minutes) per 10 days and converts this to 36 minutes over 15 days. So the sun rises approximately 18 minutes later, but we accept a clearly communicated answer of 36 minutes for this score.

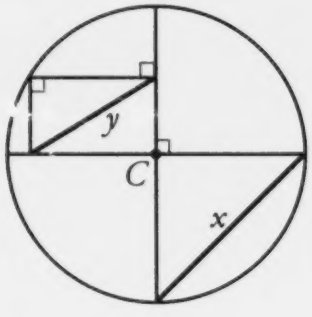
Geometry

Problem 4. The point labeled C is the center of the circle below. Right angles are as indicated. The radius of the circle is 5. Find the lengths of the thick segments labeled x and y.

This problem has a three level Rubric

The primary assessment objectives are to determine if students have basic geometric visualization skills and can utilize elementary Euclidean geometry.

0. The student cannot correctly find either value with proper justification.
1. The student explains how to correctly find one of $x = \sqrt{50} = 5\sqrt{2} = .7$ or $y = 5$. No credit without proper justification. Figures illustrating method are satisfactory. I.e. a radius drawn as the missing diagonal of the rectangle through y is enough, but the assumption that y is the diagonal of a 3-4-5 right triangle is not. For x, any form of the Pythagorean Theorem or the Isosceles Right Triangle Theorem suffices. Also, any correct use of trigonometry is satisfactory, however extremely rare.
2. The student explains how to correctly find both x and y with proper justification.



Appendix 3

The Multiple Choice ADT

CSU/UC MATHEMATICS DIAGNOSTIC TESTING PROJECT

PRECALCULUS DIAGNOSTIC TEST

A READINESS TEST FOR CALCULUS
CALCULATOR ALLOWED BUT NOT REQUIRED

PC60 – A′
1993

INSTRUCTIONS

1. Wait until you are told to start before beginning the test.
2. Work each problem and then on the answer sheet mark the space which corresponds to your answer. The test booklet, the answer sheet, and all scratch paper must be turned in when the test is finished. DO NOT WRITE IN THIS BOOKLET.
3. For each problem you are to select the best response from the given choices.
4. If you find certain questions very time consuming, leave them temporarily. Come back to them after you have gone through the entire test if you have time.
5. In scoring the test, only correct answers will be counted. If you have no idea which of the answers to a given question is the correct one, you should leave the question blank.

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PC60A93 Test Type 0116093 Precalculus

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PRECALCULUS DIAGNOSTIC TEST - 60 QUESTIONS - 90 MINUTES

1. If $a = -3$, then $|a - 1| - |3 - 2a| =$

(A) -5 (B) -1 (C) 1 (D) 5 (E) 13

2. $\left(\frac{26}{75x^4}\right)\left(\frac{3x^3}{52y}\right)(25y^2) =$

(A) $\frac{3x}{2y}$ (B) $\frac{y}{2x}$ (C) $\frac{3y}{2x}$ (D) $\frac{y^2}{2x}$ (E) $\frac{y}{6x}$

3. How many seconds does it take an object to travel 100 centimeters at an average speed of 20 centimeters per second?

(A) 0.2 (B) 5 (C) 50 (D) 80 (E) 2,000

4. If $f(x) = 3x - 4$ and $g(x) = x^2 + 1$, then $f(g(2)) =$

(A) 2 (B) 5 (C) 7 (D) 10 (E) 11

5. If $\log w = \log x + \frac{1}{2}\log y$, then $w =$

(A) $\frac{xy}{2}$ (B) $(xy)^{1/2}$ (C) $(x + y)^{1/2}$ (D) $xy^{1/2}$ (E) $x + \frac{1}{2}y$

6. If $\frac{3}{4}(x + 2) - 2 = 7x$, then $x =$

(A) $-\frac{2}{11}$ (B) 0 (C) $\frac{2}{11}$ (D) $\frac{4}{11}$ (E) $\frac{1}{2}$

GO ON TO THE NEXT PAGE.

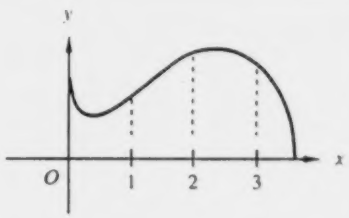
7. In the system of equations $\begin{cases} x - 3y = -1 \\ 2x + y = 12 \end{cases}$, $y =$

(A) -5 (B) -2 (C) 0 (D) 2 (E) 5

8. One of the roots of $2x^2 + 3x - 2 = 0$ is

(A) -1 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 1 (E) 2

9. The graph of the function $y = f(x)$ is shown in the figure to the right. Which of the following is true?



(A) $f(1) = f(3)$ (B) $f(2) < f(1)$

(C) $f(2) < f(3)$ (D) $f(3) < f(1)$

(E) $f(3) < f(2)$

10. The graphs of the two equations $3x + y = 2$ and $3x - 9y = 5$ are

(A) the same line (B) two perpendicular lines (C) two distinct parallel lines

(D) two intersecting lines which are not perpendicular (E) not straight lines

11. If $8^{2x+1} = 4^{1-x}$, then $x =$

(A) $-\frac{1}{8}$ (B) $-\frac{1}{5}$ (C) 0 (D) $\frac{1}{5}$ (E) $\frac{1}{8}$

12. $(8x^6y^{-3})^{1/3} =$

(A) $\frac{8x^2}{y}$ (B) $\frac{2x^2}{y}$ (C) $\frac{x^2}{2y}$ (D) $\frac{y}{2x^2}$ (E) $2x^2y$

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13. The inequality $x^2 > 4$ is equivalent to

(A) $x < -2$ (B) $x > 2$ (C) $-2 < x < 2$

(D) $x < 0$ or $x > 2$ (E) $x < -2$ or $x > 2$

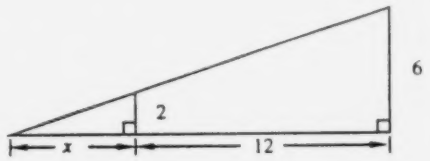
14. When $2x^2 - 7x + 4$ is divided by $x - 3$ the remainder is

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

15. In the figure shown to the right, $x =$

(A) 2 (B) 4 (C) 6

(D) 9 (E) 10



16. The perimeter of a square is directly proportional to the length of a side and its area is directly proportional to the square of the length of a side. If the perimeter of a square is multiplied by 3, then its area is multiplied by

(A) $\frac{1}{9}$ (B) $\frac{1}{3}$ (C) 3 (D) 6 (E) 9

17. $\frac{2w-1}{w+1} - \frac{1}{w-1} =$

(A) $\frac{2w(w-2)}{w^2-2w+1}$ (B) $\frac{2w(w-2)}{w^2-1}$ (C) $\frac{2w}{w^2-1}$

(D) $\frac{2(w-1)}{w+1}$ (E) $\frac{2}{w+1}$

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18. $\sqrt{16x^6 + 4a^2x^4} =$

(A) $4x^3 + 2ax^2$ (B) $4x^4 + 2ax^2$ (C) $2x^2\sqrt{4x^2 + a^2}$

(D) $2ax^2\sqrt{4x^2 + 1}$ (E) $4x^2\sqrt{4x^2 + a^2}$

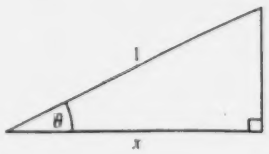
19. If a and b are in the domain of a function f and $f(a) < f(b)$, which of the following must be true?

(A) $a = 0$ or $b = 0$ (B) $a < b$ (C) $a \leq b$ (D) $a \neq b$ (E) $a = b$

20. In the figure shown to the right, $\sin\theta =$

(A) $\sqrt{1 + x^2}$ (B) $\dfrac{\sqrt{1 + x^2}}{x}$ (C) x

(D) $\sqrt{1 - x^2}$ (E) $\dfrac{\sqrt{1 - x^2}}{x}$

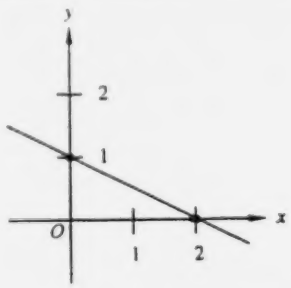


21. An equation of the line in the figure shown to the right is

(A) $\frac{1}{2}x + y = 1$ (B) $x + \frac{1}{2}y = 1$

(C) $x + y = 1$ (D) $x + 2y = 1$

(E) $2x + y = 1$



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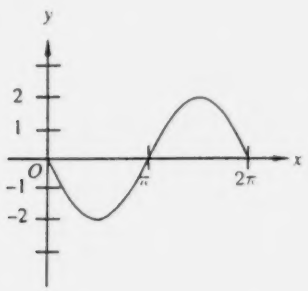
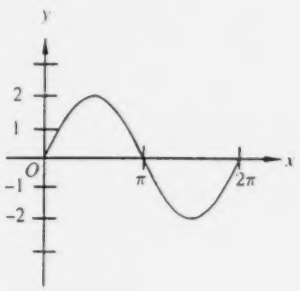
22. Which of the following could be a portion of the graph of $y = -2\sin x$?

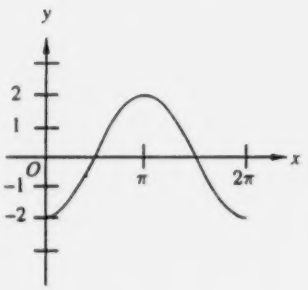
(A)



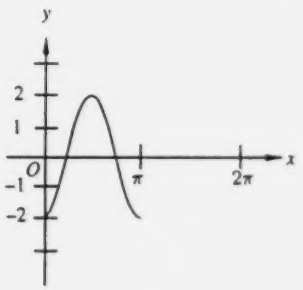
(B)



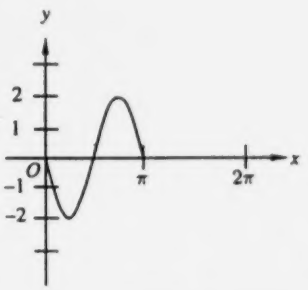
(C)



(D)



(E)



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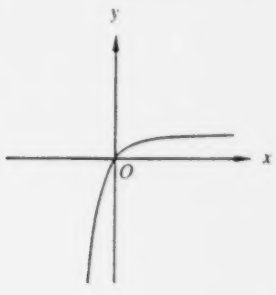
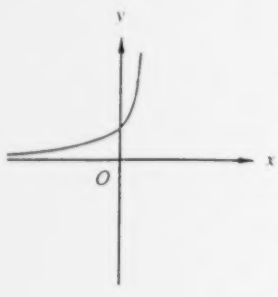
23. Which of the following could be a portion of the graph of $y = \log_6 x$?

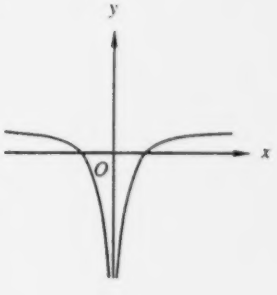
(A)



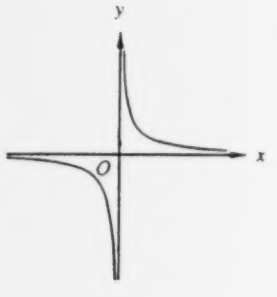
(B)



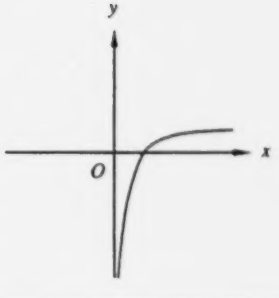
(C)



(D)



(E)



GO ON TO THE NEXT PAGE

24. One of the roots of $x^2 + x - 3 = 0$ is

(A) $\frac{1+\sqrt{13}}{2}$ (B) $\frac{1-2\sqrt{3}}{2}$ (C) $\frac{-1+\sqrt{3}}{2}$

(D) $\frac{-1-\sqrt{13}}{2}$ (E) $\frac{-1-2\sqrt{3}}{2}$

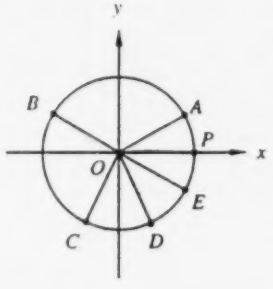
25. What number must be added to $x^2 + 3x$ to form a perfect square?

(A) $\frac{3}{2}$ (B) $\frac{9}{2}$ (C) $\frac{9}{4}$ (D) 6 (E) 9

26. In the figure shown to the right, if $\cos\theta = -\frac{5}{6}$, which angle could best represent θ?

(A) $\angle POA$ (B) $\angle POB$ (C) $\angle POC$

(D) $\angle POD$ (E) $\angle POE$

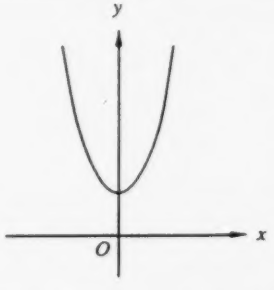


27. Which of the following could be an equation for the graph shown in the figure to the right?

(A) $y = x^2 + 3$ (B) $y = x^2 - 3$

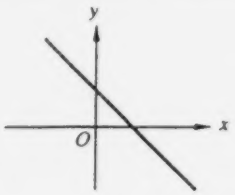
(C) $y = (x-3)^2$ (D) $y = 3 - x^2$

(E) $x = y^2 + 3$

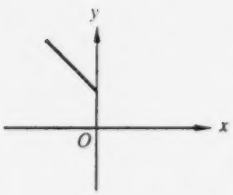


GO ON TO THE NEXT PAGE.

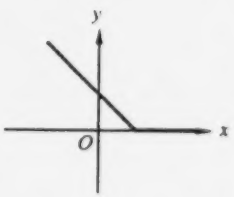
28. The graph of $y = f(x)$ is shown in the figure to the right. Which of the following is the graph of $y = |f(x)|$?



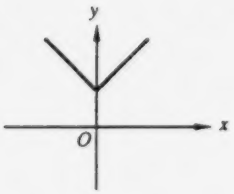
(A)



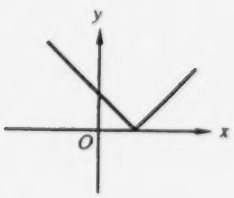
(B)



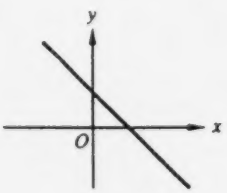
(C)



(D)

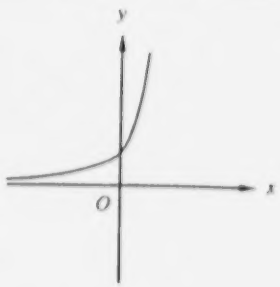


(E)



GO ON TO THE NEXT PAGE.

29. Which of the following could be an equation for the graph shown in the figure to the right?



(A) $y = x^2$ (B) $y = 2^x - 1$

(C) $y = 2^x$ (D) $y = 2^{-x}$

(E) $y = \log_2 x$

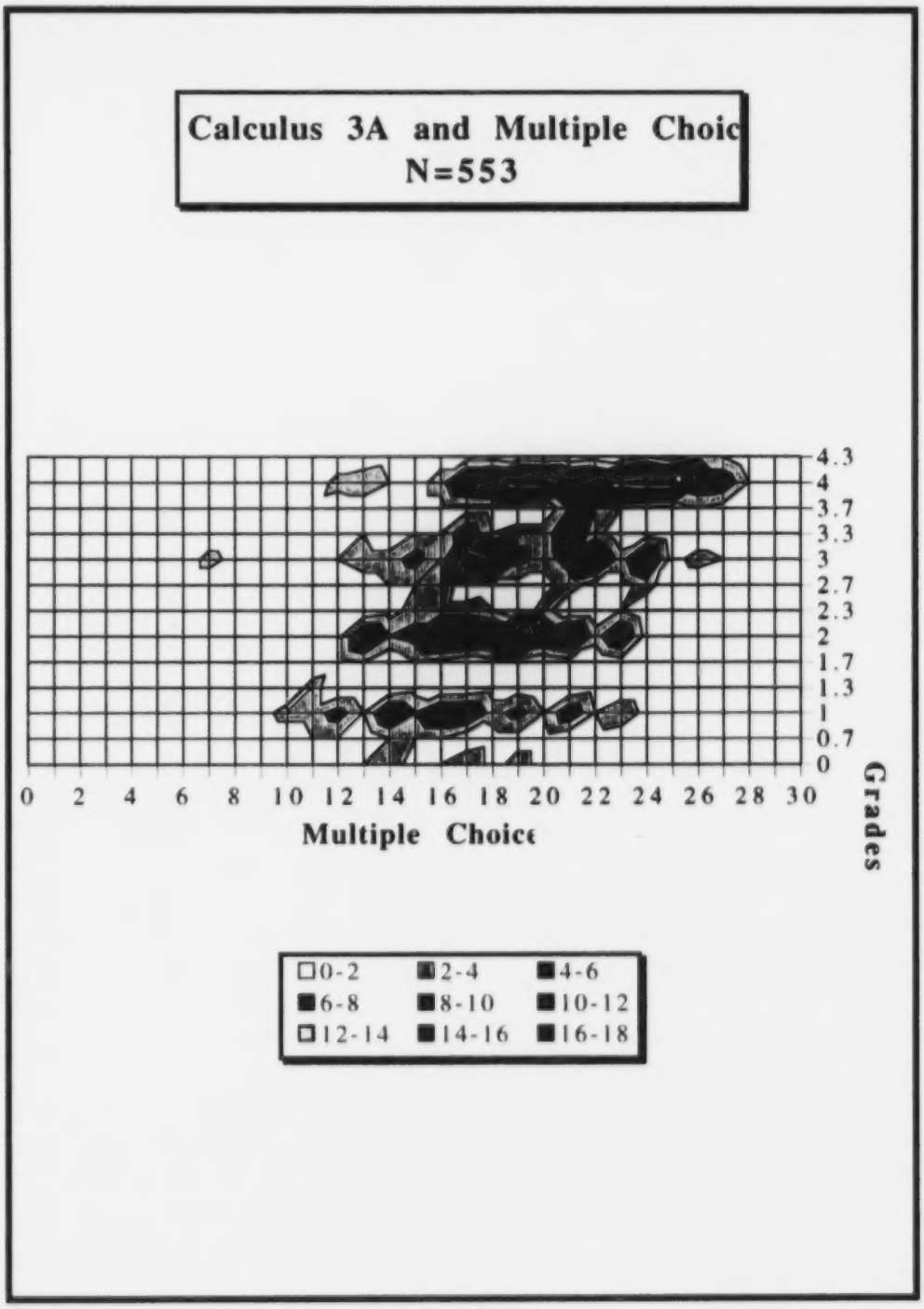
30. If $f(x) = x + 3$ and $f(a + 1) = 2f(a)$, then $a =$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 3

END OF EXAMINATION

Appendix 4

Data Scatter Plots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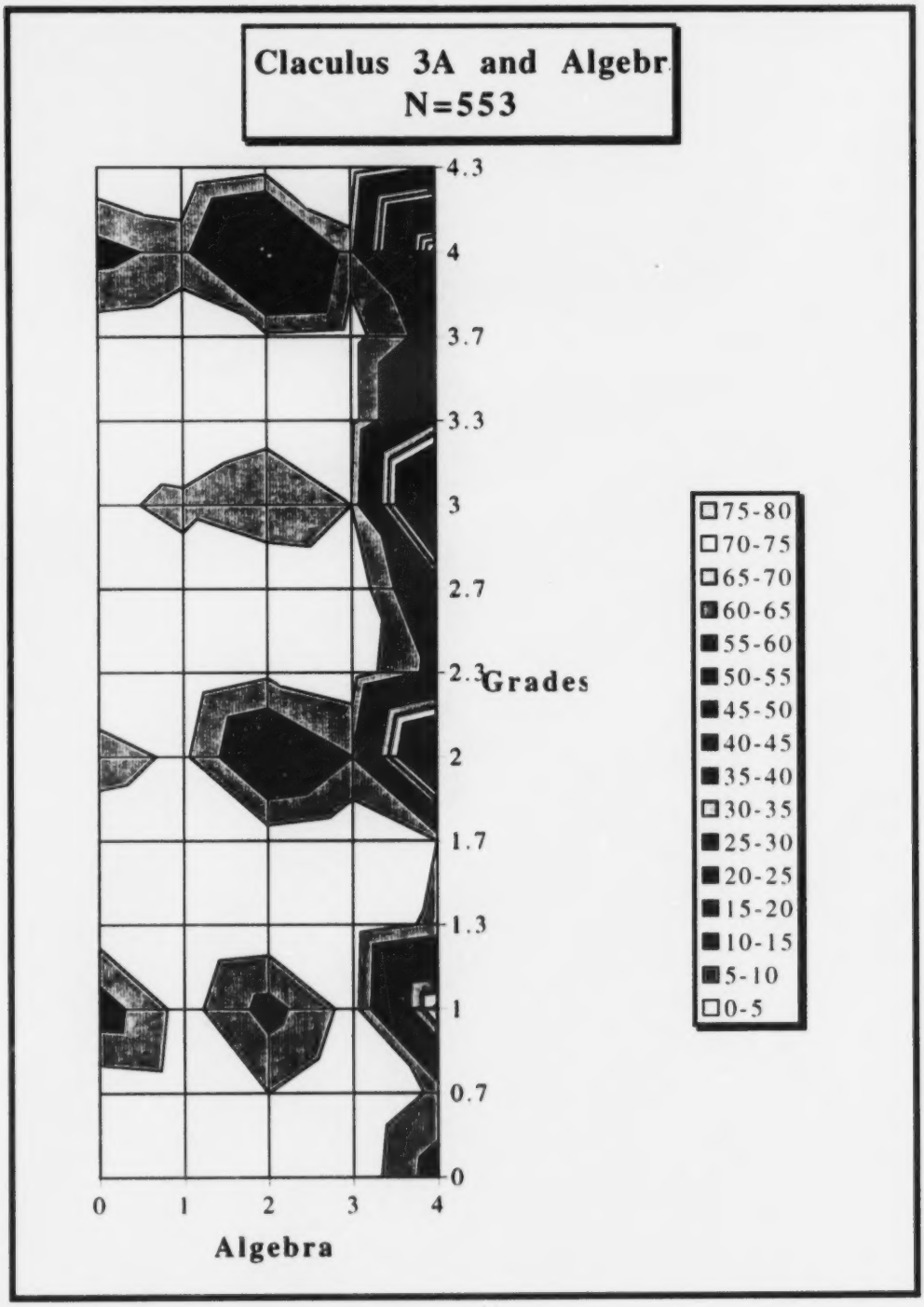
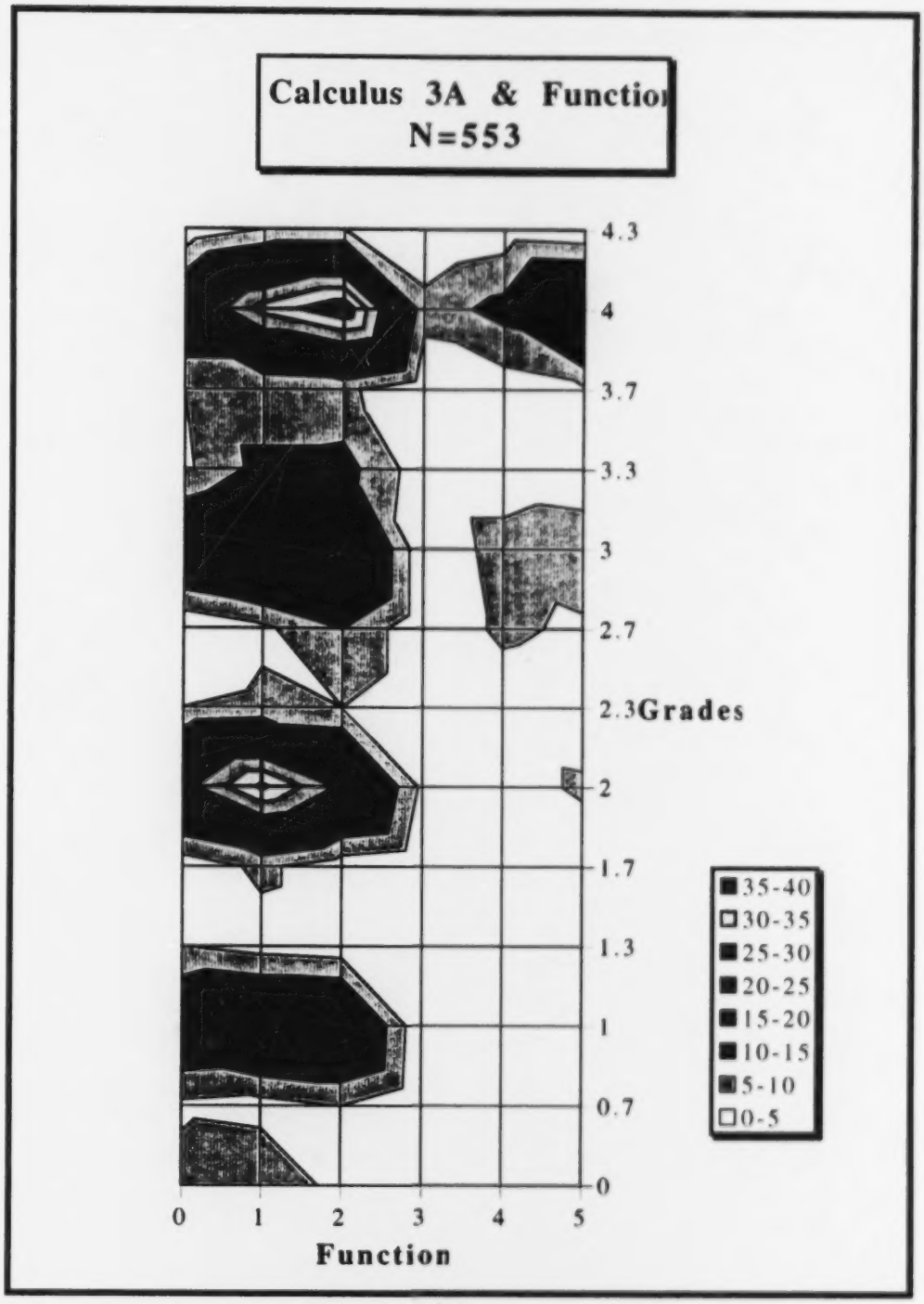
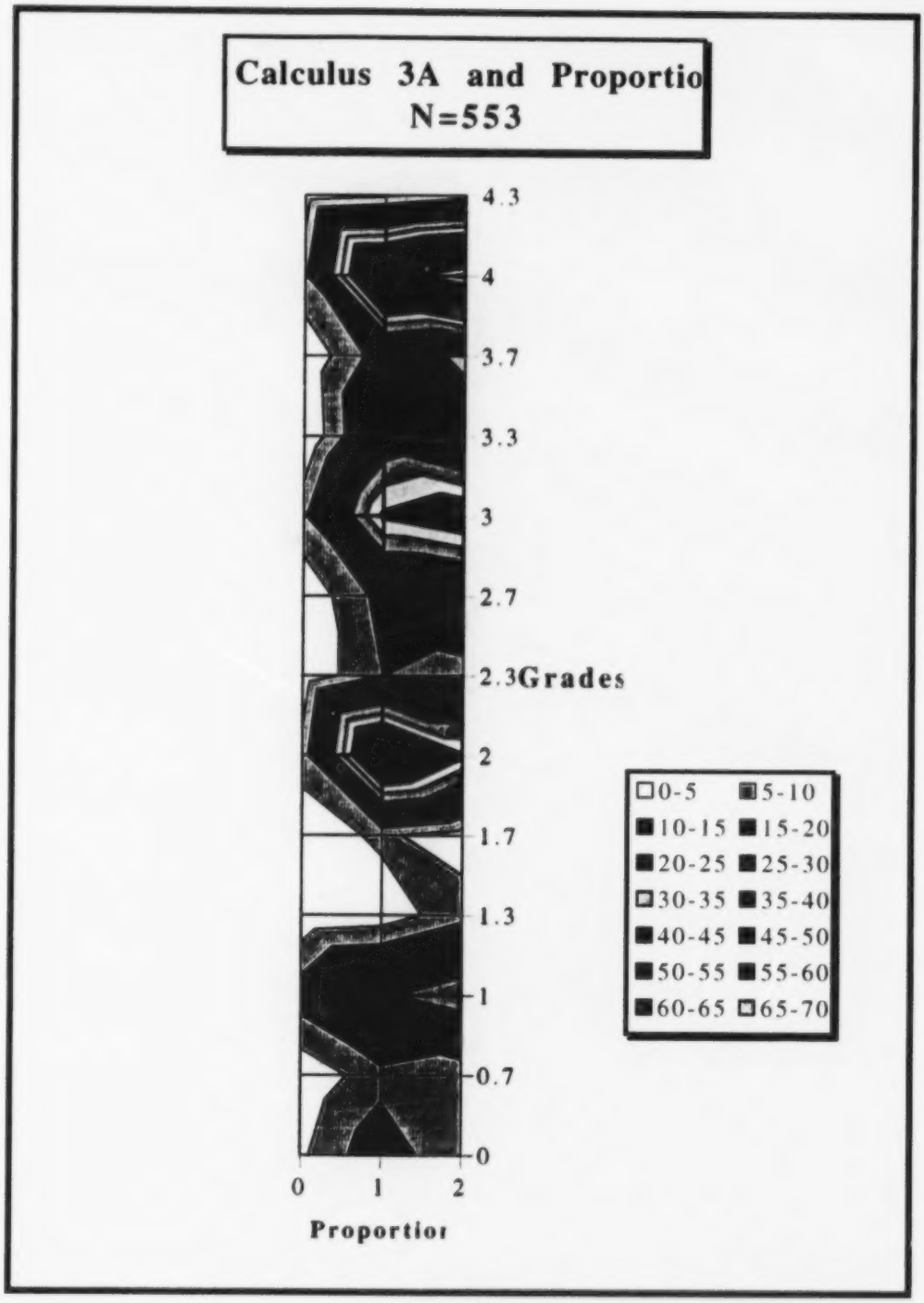
4.3
4
3.7
3.3
3
2.7
2.3
2
1.7
1.3
1
0.7
0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
Multiple Choice
Grades
0-2
2-4
4-6
6-8
8-10
10-12
12-14
14-16
16-18


Claculus 3A and Algebr
N=553
4.3
4
3.7
3.3
3
2.7
2.3
2
1.7
1.3
1
0.7
0
Grades
0
1
2
3
4
Algebra
75-80
70-75
65-70
60-65
55-60
50-55
45-50
40-45
35-40
30-35
25-30
20-25
15-20
10-15
5-10
0-5

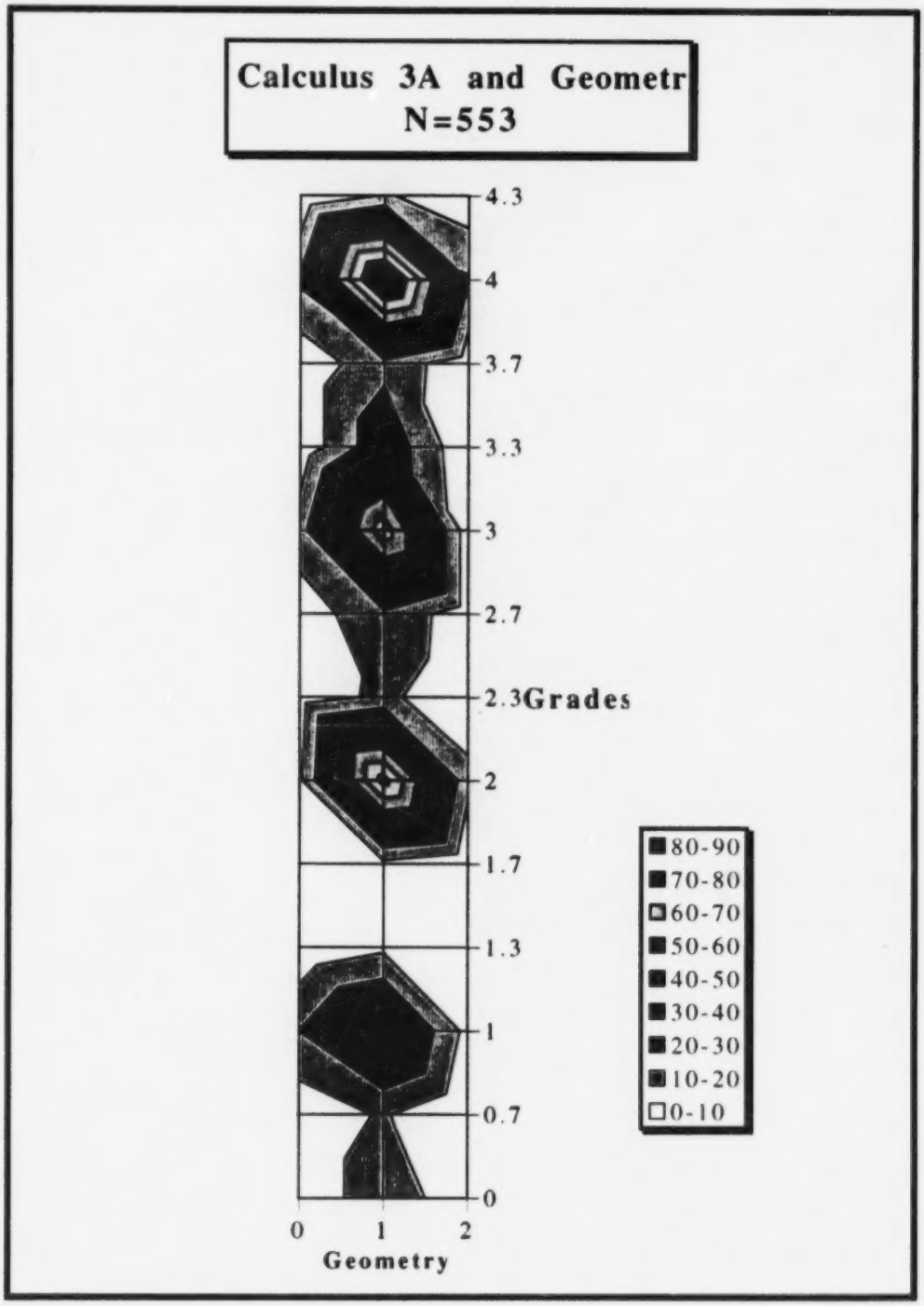
39


Calculus 3A & Functio
N=553
4.3
4
3.7
3.3
3
2.7
2.3
2
1.7
1.3
1
0.7
0
Grades
0
1
2
3
4
5
Function
35-40
30-35
25-30
20-25
15-20
10-15
5-10
0-5

40


Calculus 3A and Proportio
N=553
4.3
4
3.7
3.3
3
2.7
2.3
2
1.7
1.3
1
0.7
0
Grades
0
1
2
Proportior
0-5
5-10
10-15
15-20
20-25
25-30
30-35
35-40
40-45
45-50
50-55
55-60
60-65
65-70

41


Calculus 3A and Geometr
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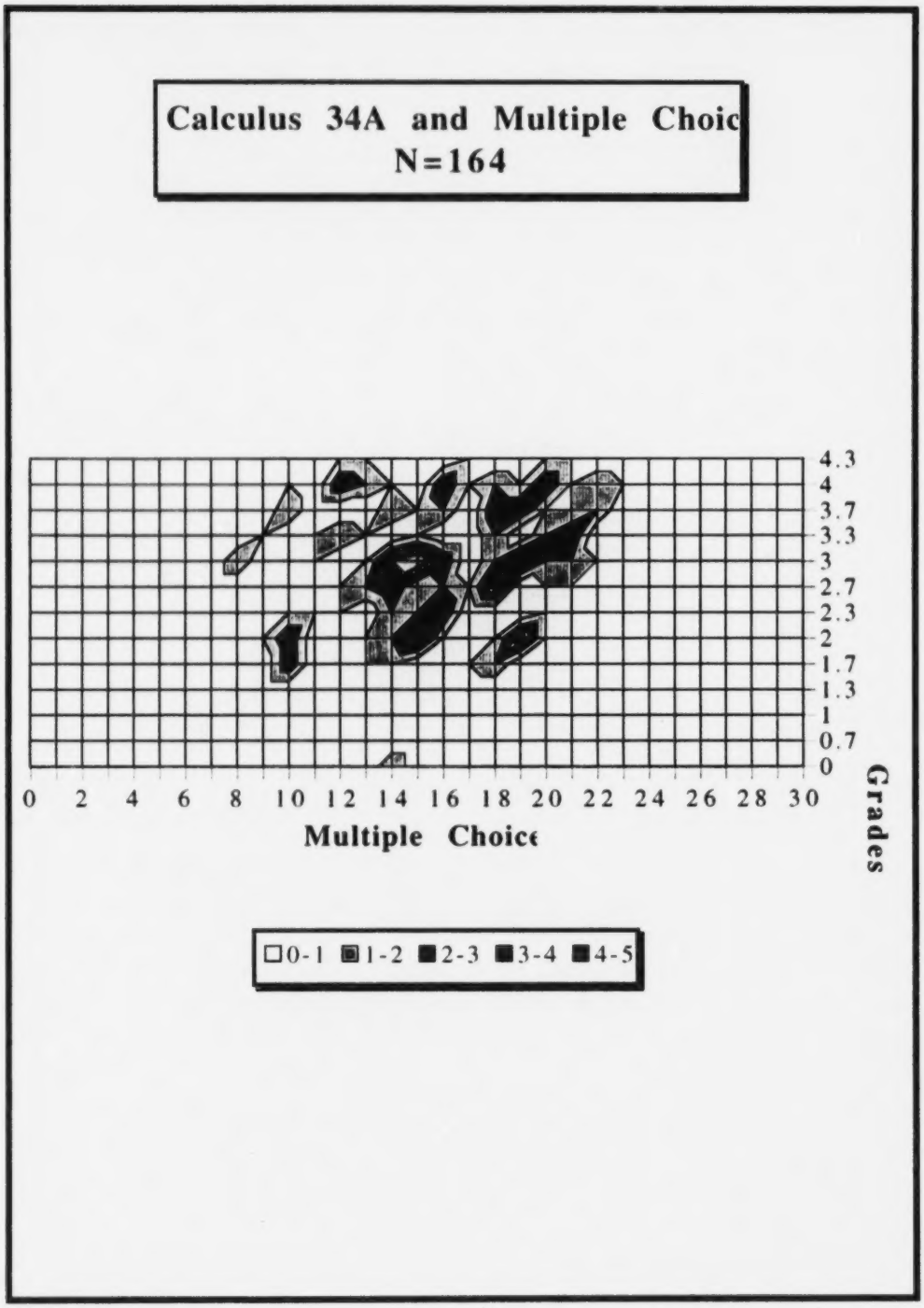
4.3
4
3.7
3.3
3
2.7
2.3
2
1.7
1.3
1
0.7
0
Grades
0
1
2
Geometry
80-90
70-80
60-70
50-60
40-50
30-40
20-30
10-20
0-10

42


Calculus 34A and Multiple Choic
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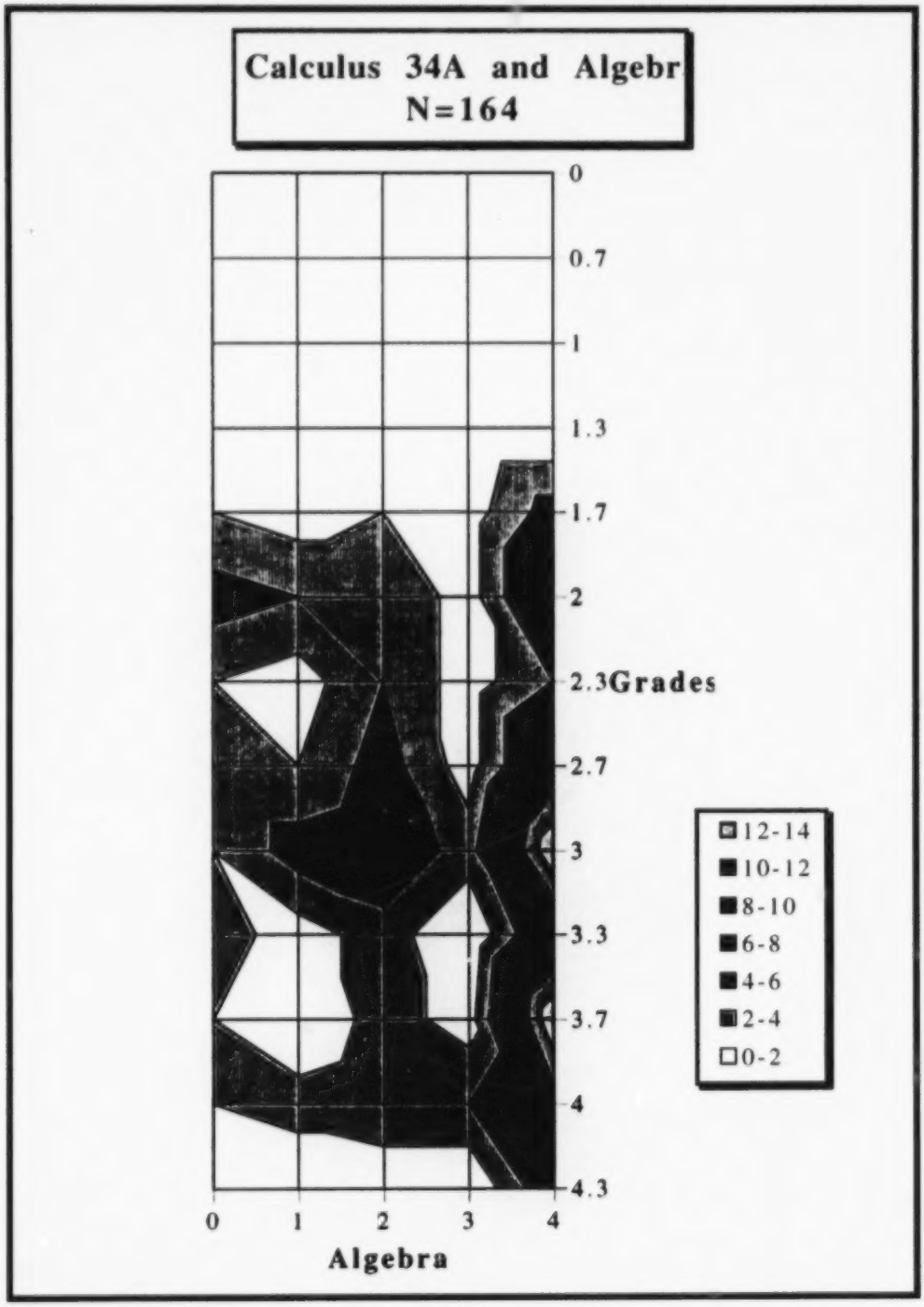
4
3.7
3.3
3
2.7
2.3
2
1.7
1.3
1
0.7
0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30
Multiple Choice
Grades
0-1 1-2 2-3 3-4 4-5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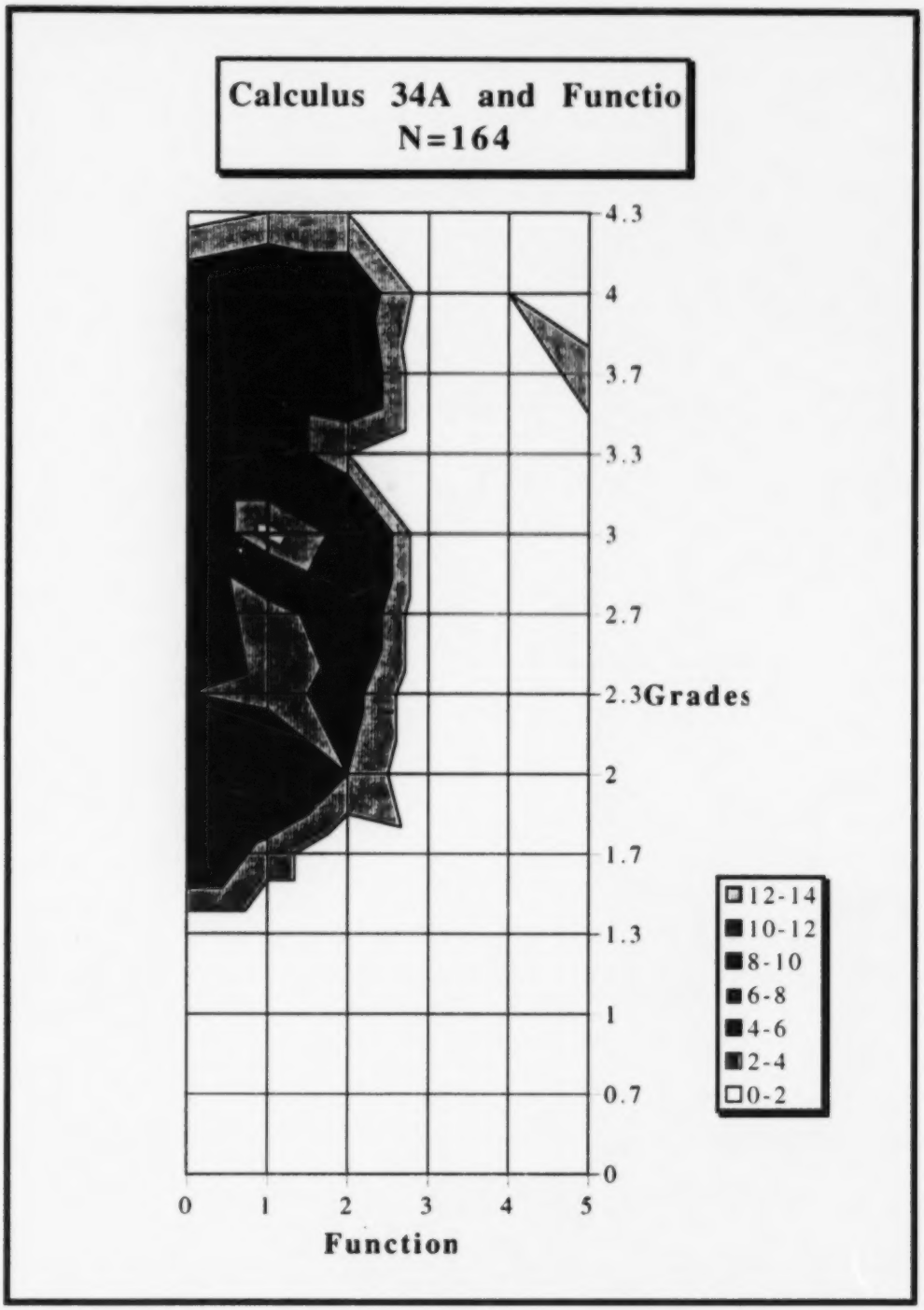
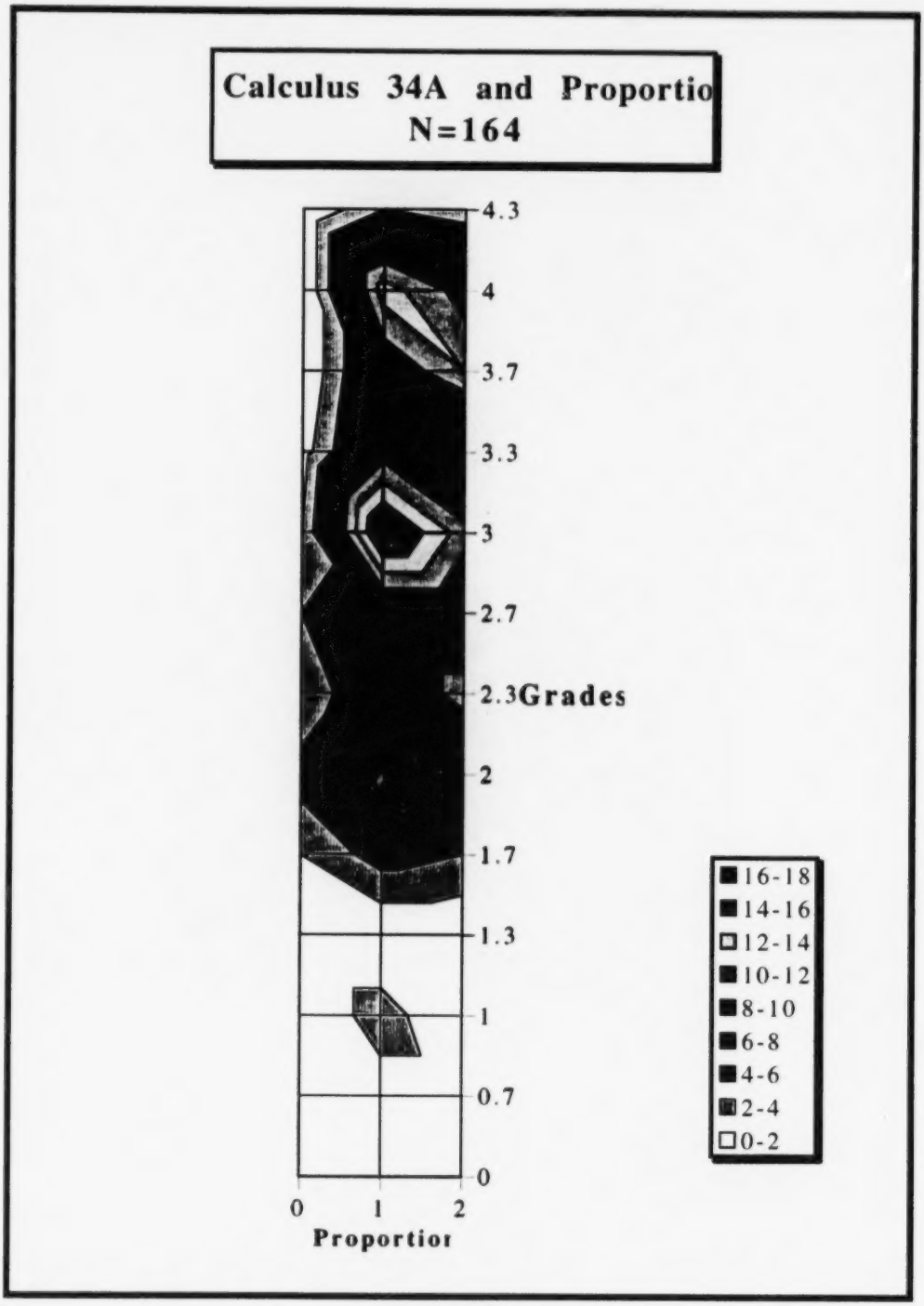
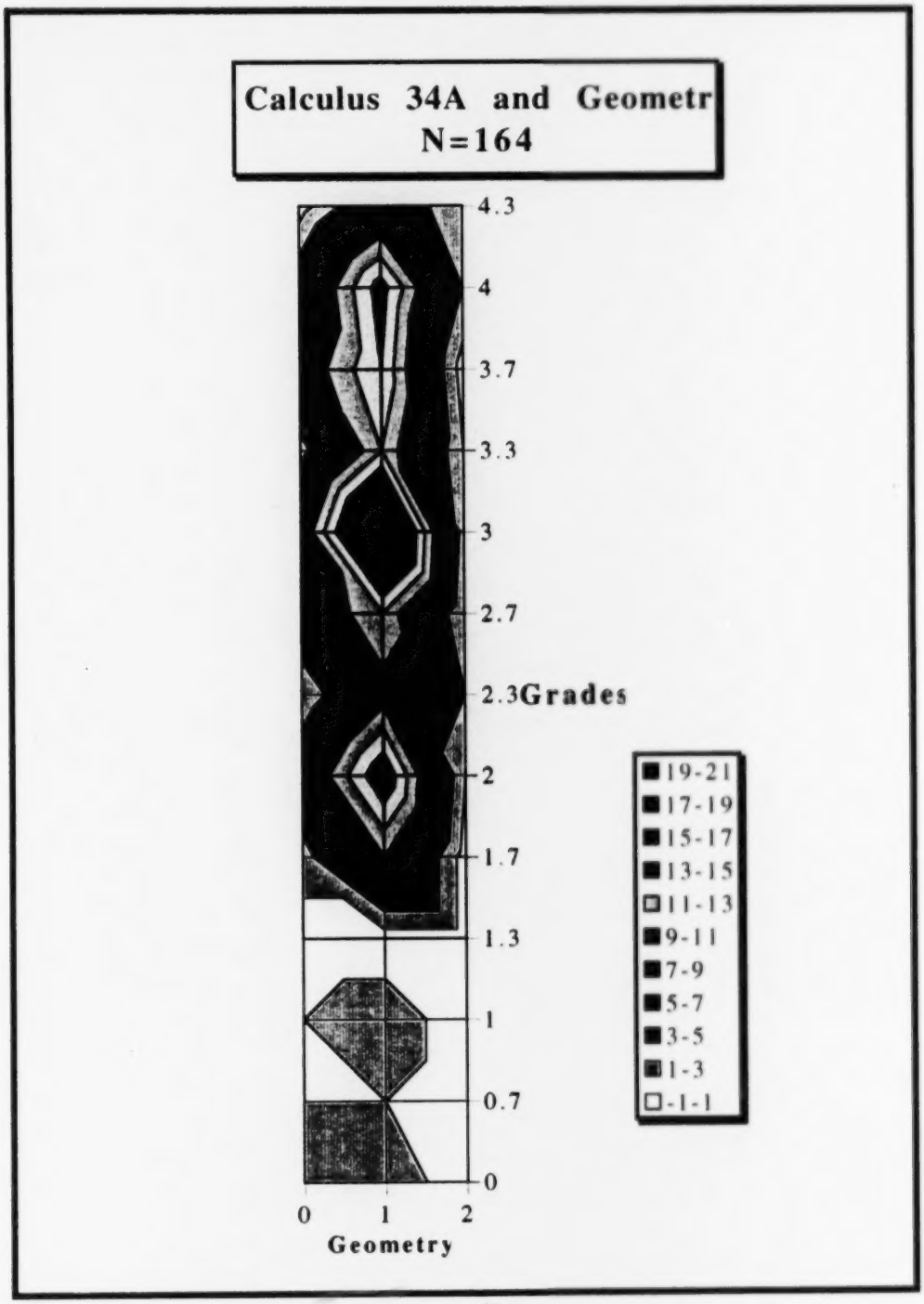
0
0.7
1
1.3
1.7
2
2.3
2.7
3
3.3
3.7
4
4.3
Grades
0
1
2
3
4
Algebra
12-14
10-12
8-10
6-8
4-6
2-4
0-2

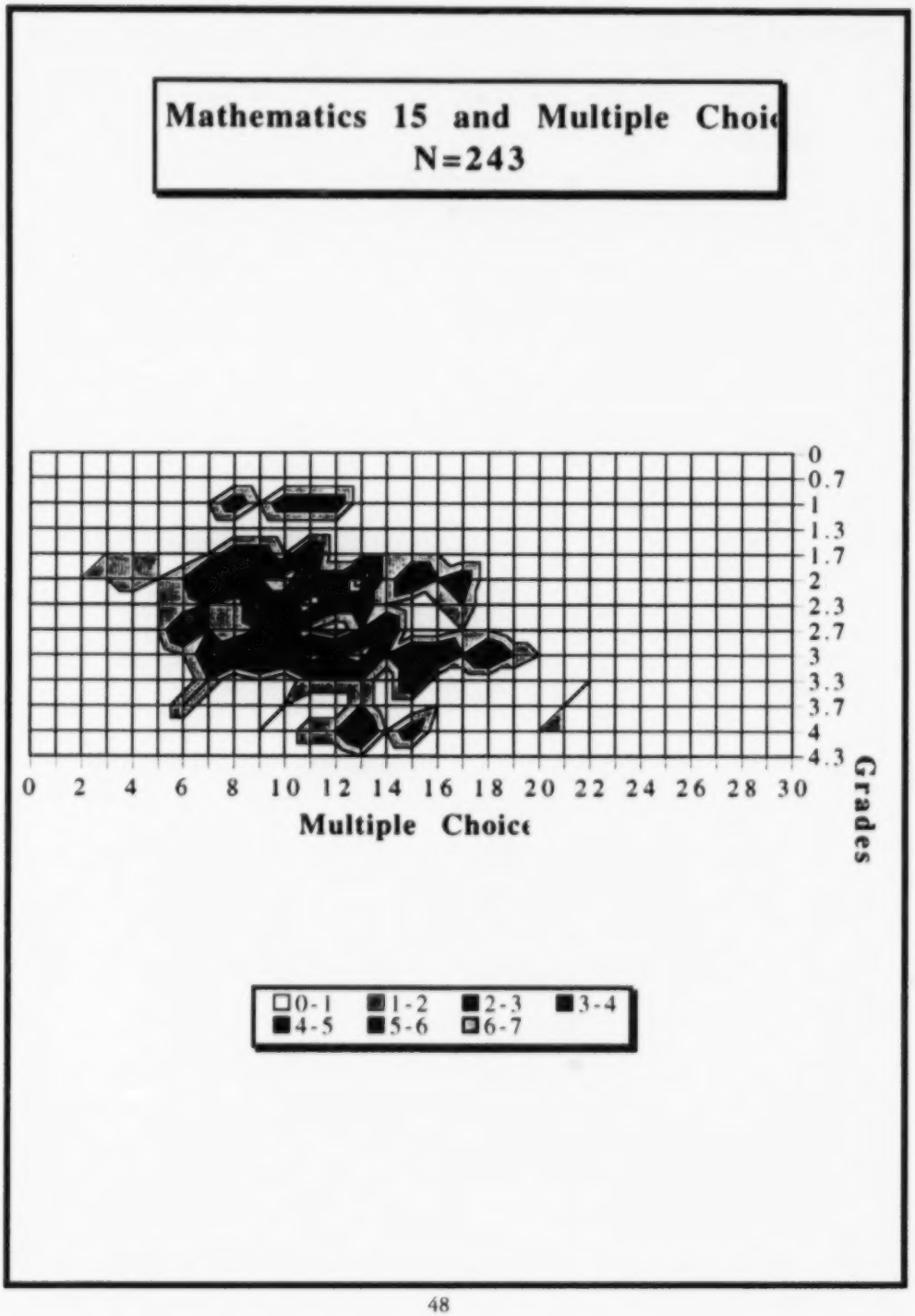
44


Calculus 34A and Functio
N=164
4.3
4
3.7
3.3
3
2.7
2.3
2
1.7
1.3
1
0.7
0
Grades
12-14
10-12
8-10
6-8
4-6
2-4
0-2
0
1
2
3
4
5
Function


Calculus 34A and Proportio
N=164
4.3
4
3.7
3.3
3
2.7
2.3Grades
2
1.7
1.3
1
0.7
0
0
1
2
Proportior
16-18
14-16
12-14
10-12
8-10
6-8
4-6
2-4
0-2


Calculus 34A and Geometr
N=164
4.3
4
3.7
3.3
3
2.7
2.3
2
1.7
1.3
1
0.7
0
Grades
19-21
17-19
15-17
13-15
11-13
9-11
7-9
5-7
3-5
1-3
-1-1
0
1
2
Geometry

47


Mathematics 15 and Multiple Choi
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0
0.7
1
1.3
1.7
2
2.3
2.7
3
3.3
3.7
4
4.3
Grades
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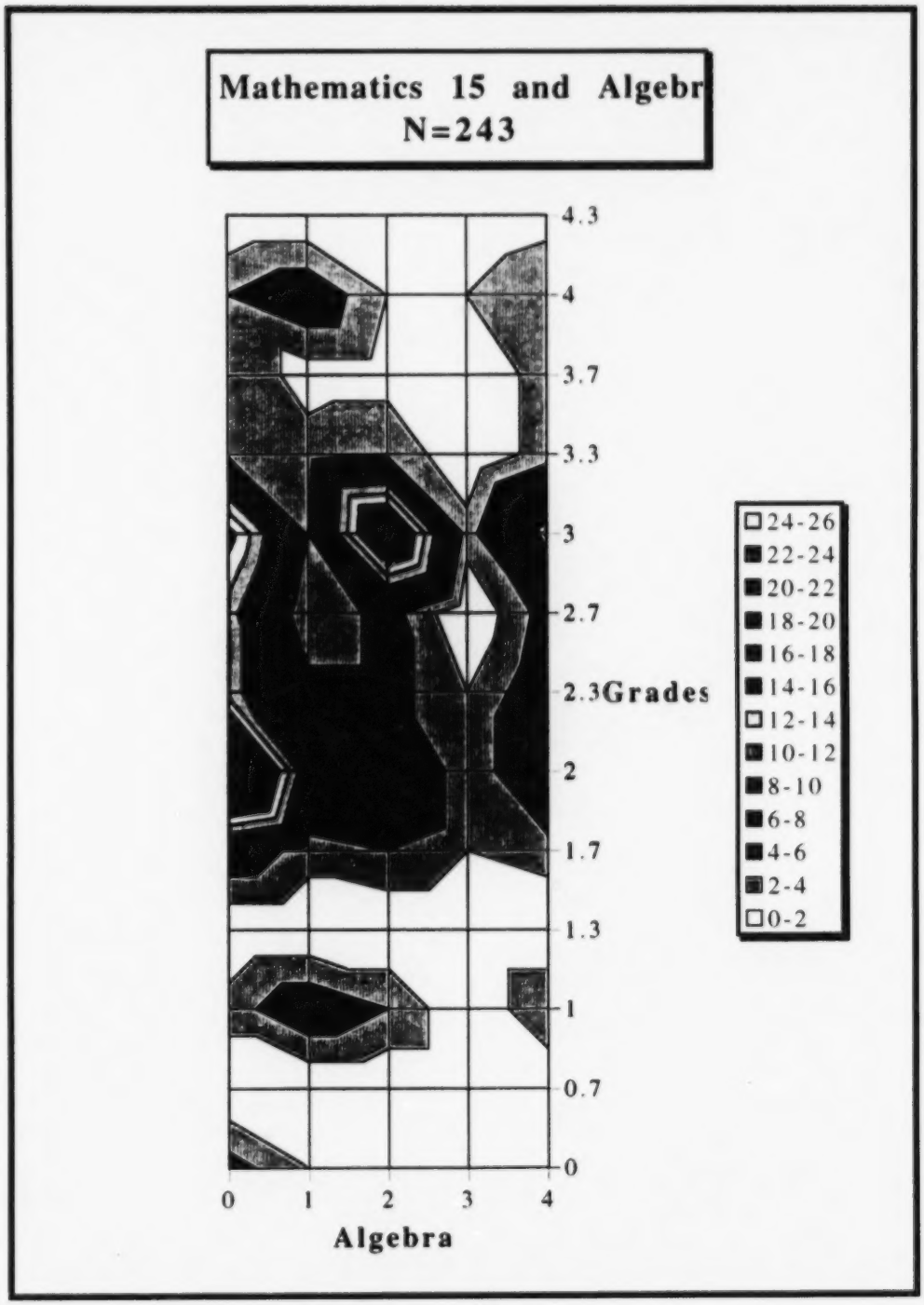
Multiple Choice
0-1
1-2
2-3
3-4
4-5
5-6
6-7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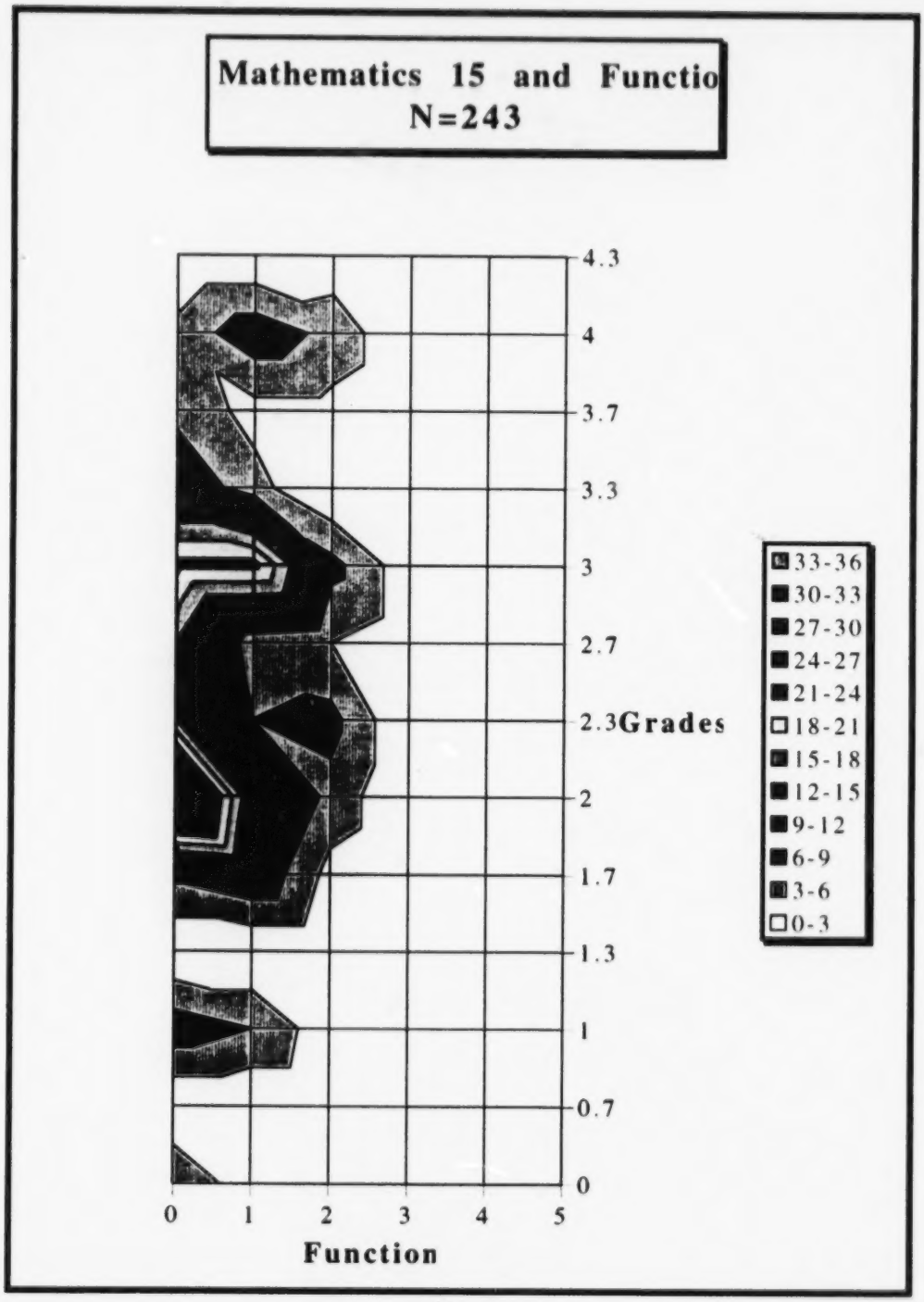
3.7
3.3
3
2.7
2.3Grades
2
1.7
1.3
1
0.7
0
0
1
2
3
4
Algebra
24-26
22-24
20-22
18-20
16-18
14-16
12-14
10-12
8-10
6-8
4-6
2-4
0-2


Mathematics 15 and Functio
N=243
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3.7
3.3
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2.7
2.3
2
1.7
1.3
1
0.7
0
Grades
33-36
30-33
27-30
24-27
21-24
18-21
15-18
12-15
9-12
6-9
3-6
0-3
0
1
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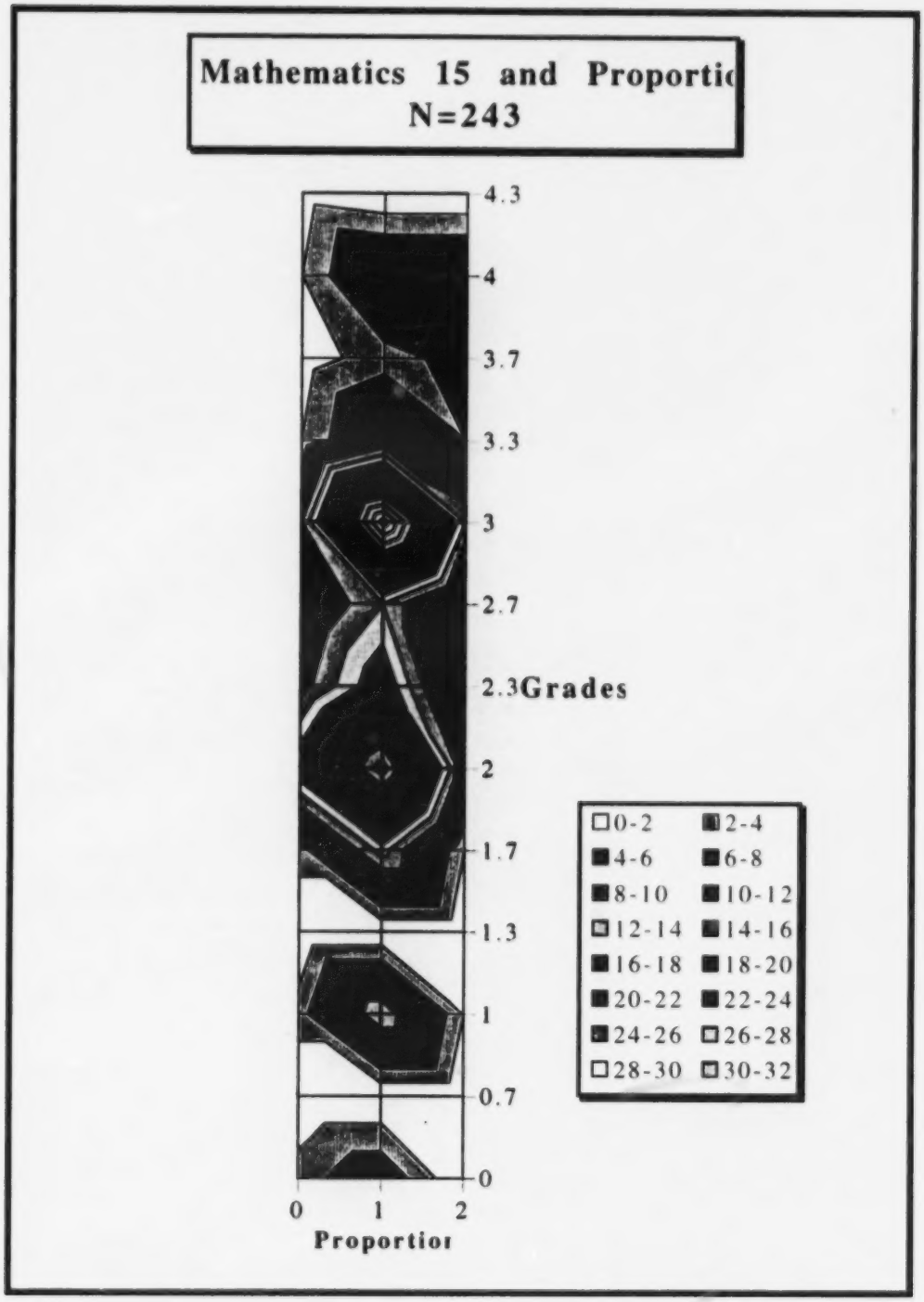
3
4
5
Function

50


Mathematics 15 and Proportic
N=243
4.3
4
3.7
3.3
3
2.7
2.3Grades
2
1.7
1.3
1
0.7
0
0
1
2
Proportior
0-2
2-4
4-6
6-8
8-10
10-12
12-14
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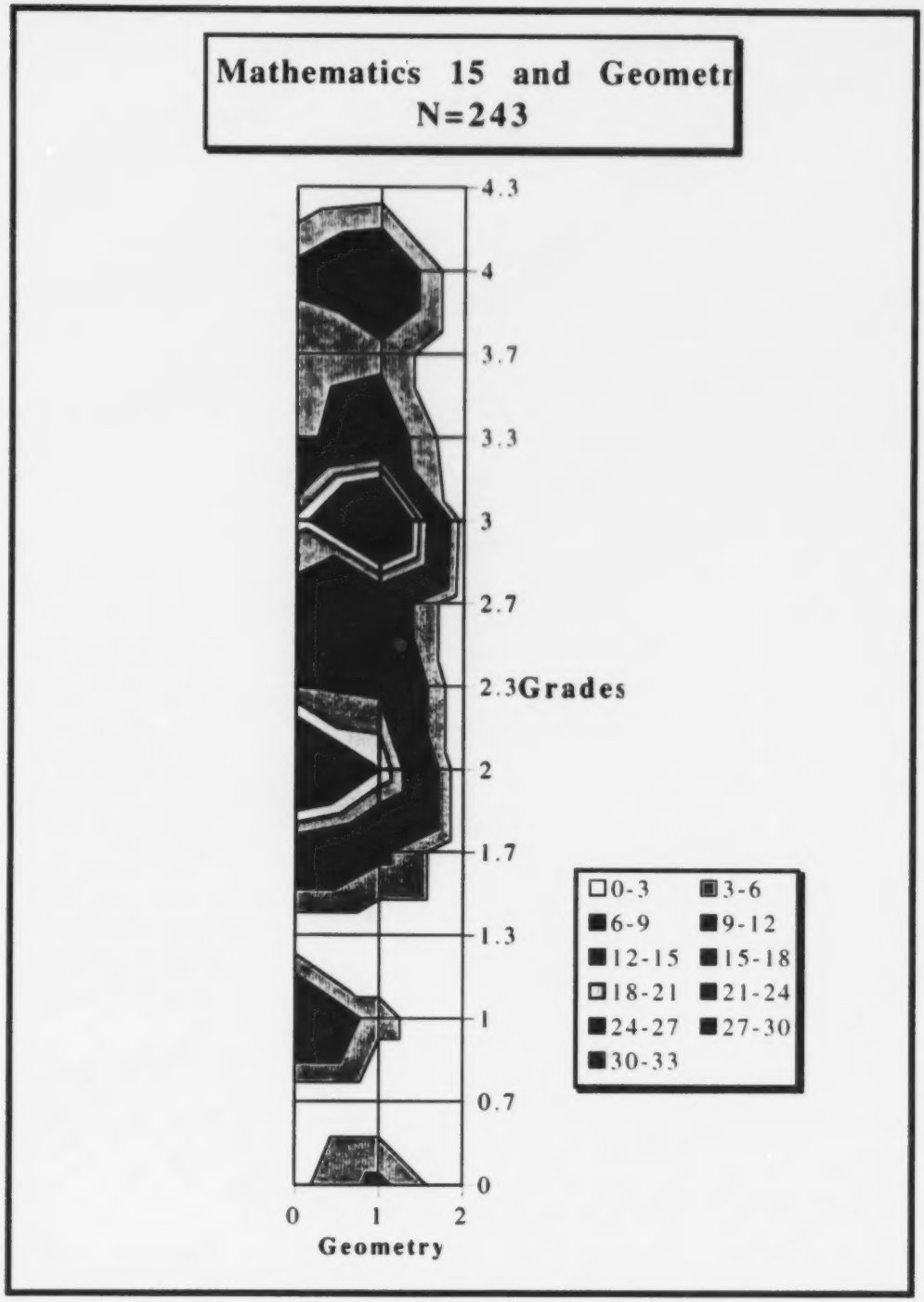
16-18
18-20
20-22
22-24
24-26
26-28
28-30
30-32

51


Mathematics 15 and Geometr
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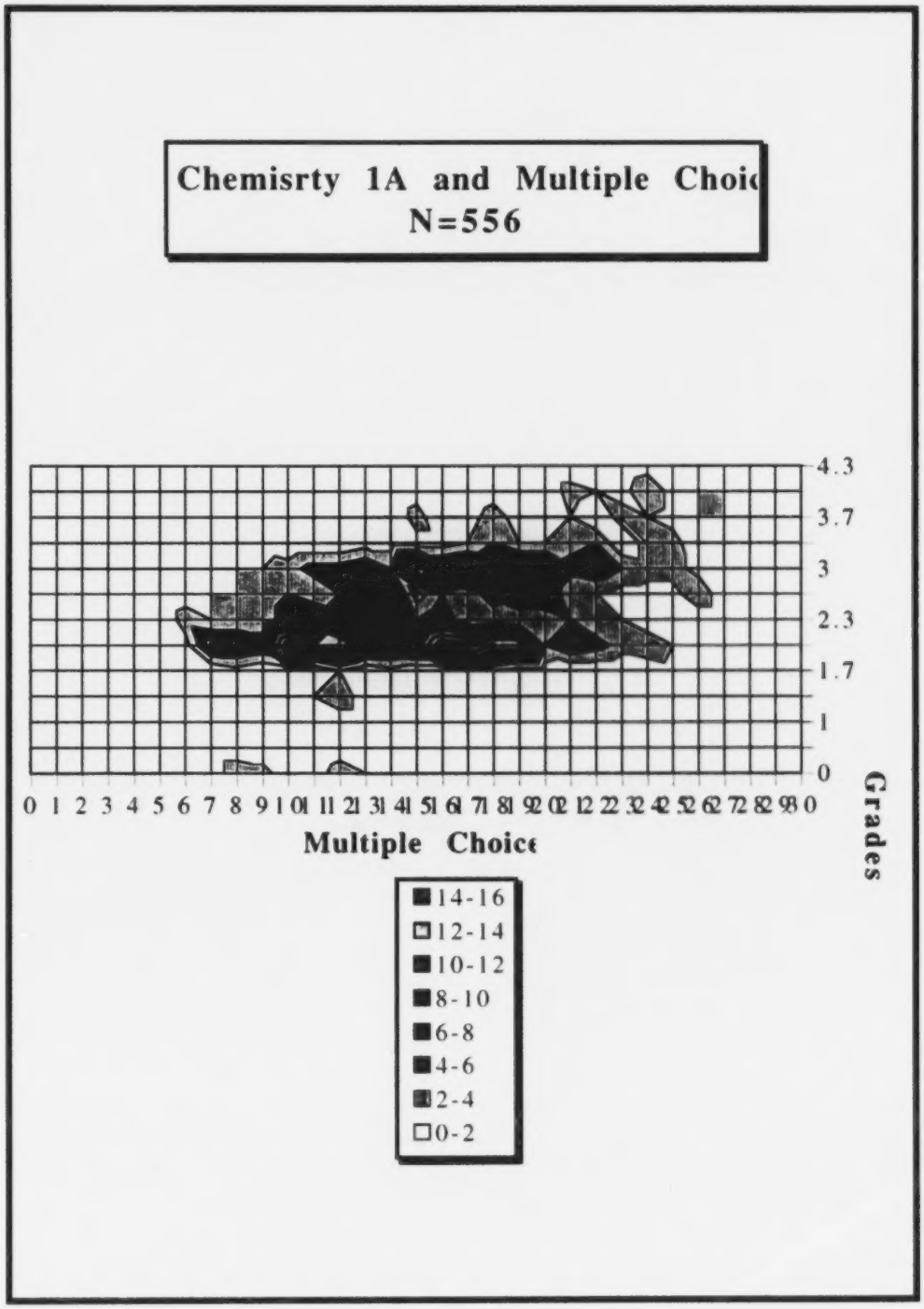
4.3
4
3.7
3.3
3
2.7
2.3
Grades
2
1.7
1.3
1
0.7
0
0
1
2
Geometry
0-3
3-6
6-9
9-12
12-15
15-18
18-21
21-24
24-27
27-30
30-33

52


Chemisrty 1A and Multiple Choic
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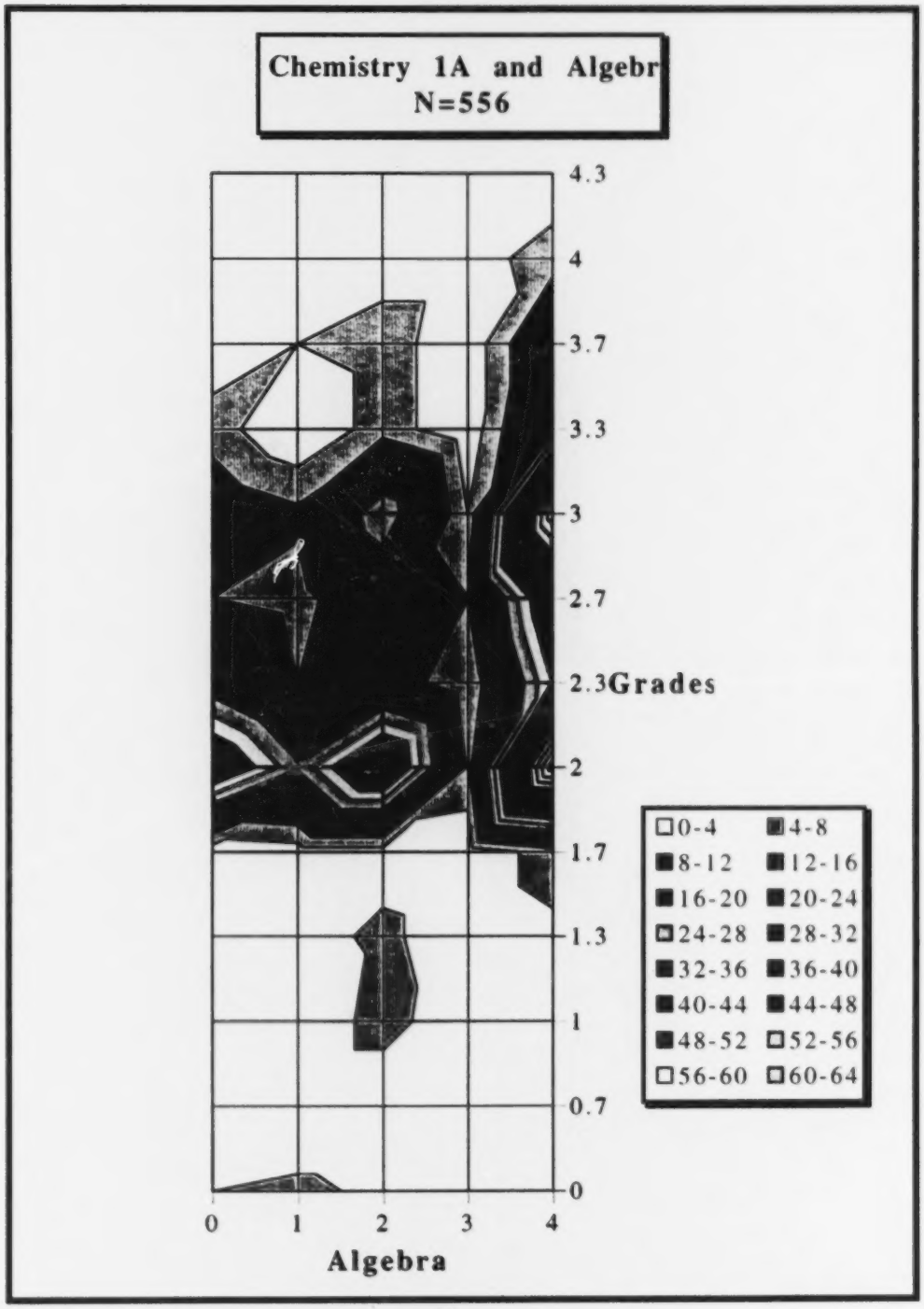
4.3
3.7
3
2.3
1.7
1
0
Grades
Multiple Choice
14-16
12-14
10-12
8-10
6-8
4-6
2-4
0-2


Chemistry 1A and Algebr
N=556
4.3
4
3.7
3.3
3
2.7
2.3
Grades
2
1.7
1.3
1
0.7
0
0
1
2
3
4
Algebra
0-4
4-8
8-12
12-16
16-20
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28-32
32-36
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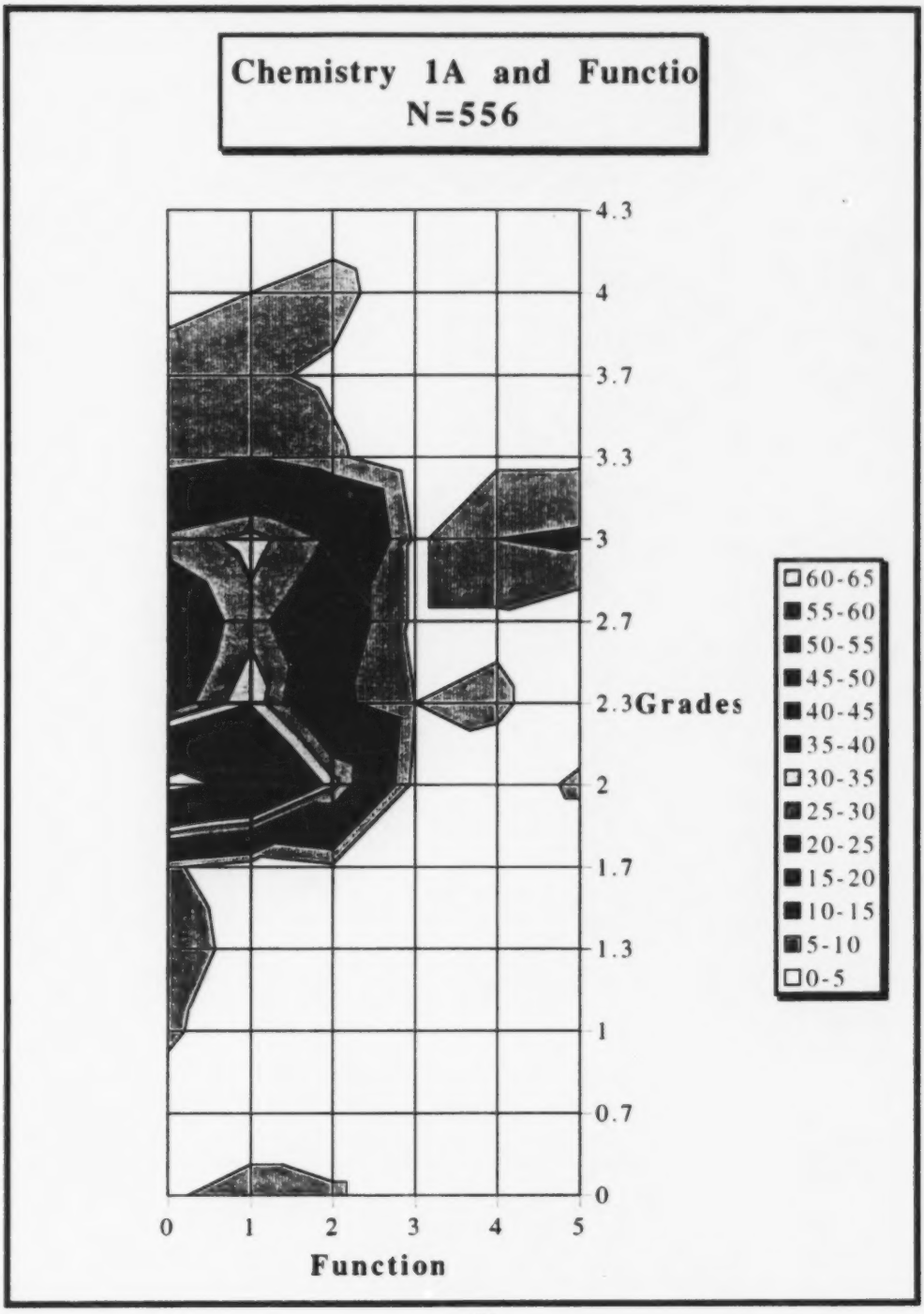
56-60
60-64


Chemistry 1A and Functio
N=556
4.3
4
3.7
3.3
3
2.7
2.3
2
1.7
1.3
1
0.7
0
Grades
60-65
55-60
50-55
45-50
40-45
35-40
30-35
25-30
20-25
15-20
10-15
5-10
0-5
0
1
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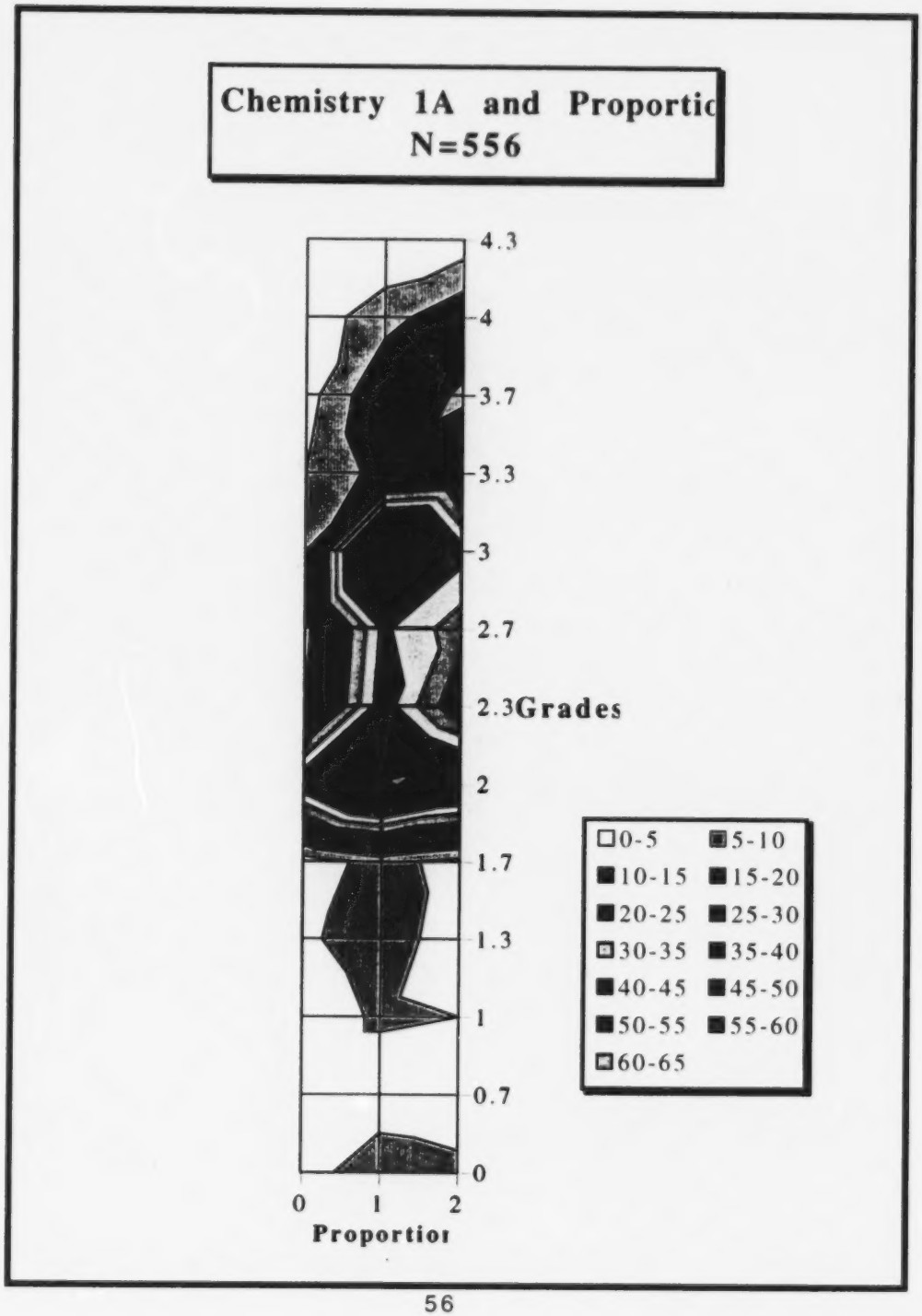
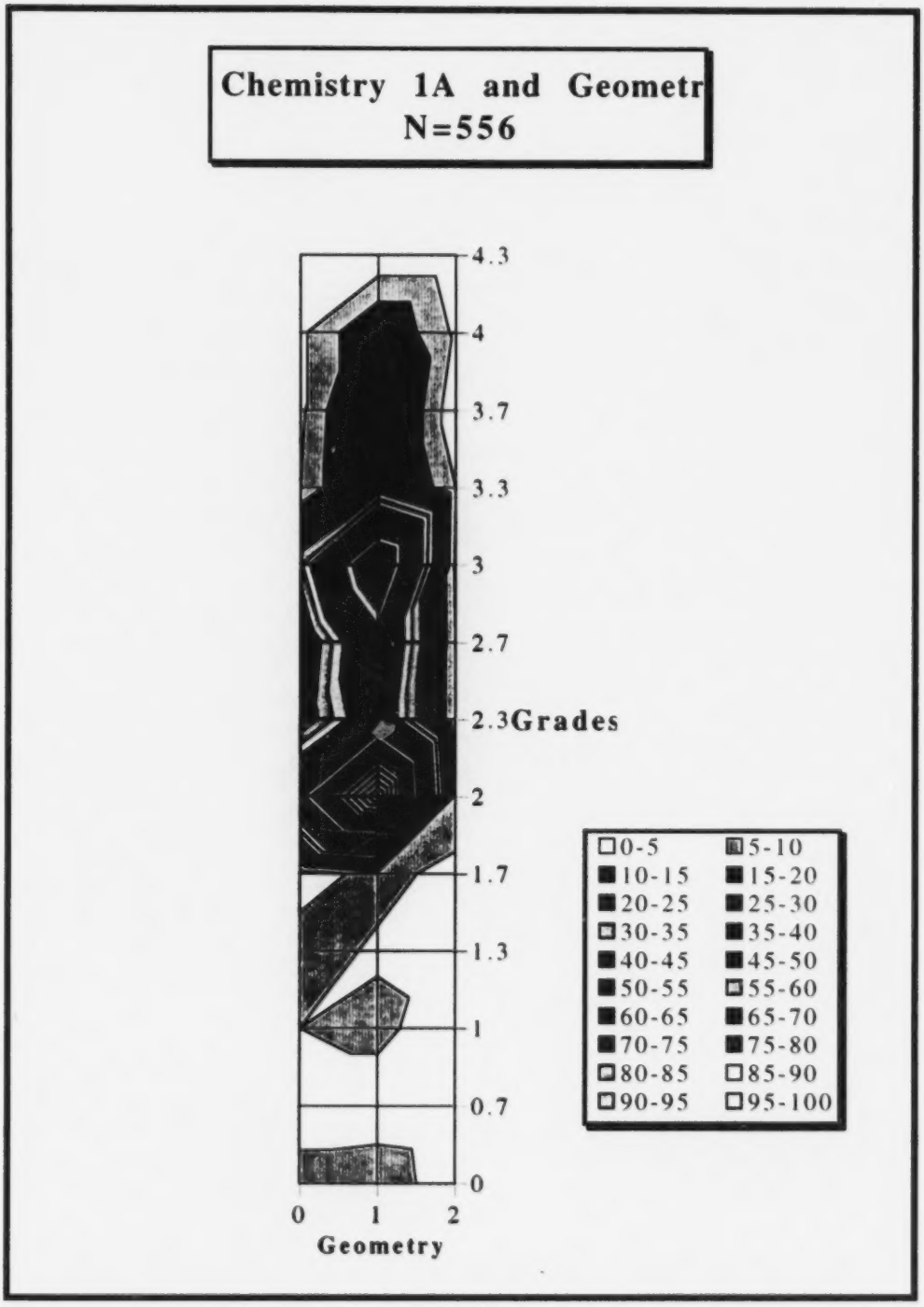
3
4
5
Function

55


Chemistry 1A and Proportic
N=556
4.3
4
3.7
3.3
3
2.7
2.3
Grades
2
1.7
1.3
1
0.7
0
0
1
2
Proportio
0-5
5-10
10-15
15-20
20-25
25-30
30-35
35-40
40-45
45-50
50-55
55-60
60-65

56


Chemistry 1A and Geometr
N=556
4.3
4
3.7
3.3
3
2.7
2.3
Grades
2
1.7
1.3
1
0.7
0
0
1
2
Geometry
0-5
5-10
10-15
15-20
20-25
25-30
30-35
35-40
40-45
45-50
50-55
55-60
60-65
65-70
70-75
75-80
80-85
85-90
90-95
95-100

Appendix 5

Tables

TABLE 1
ADT
N=1733

Final Recorded Score (60 Points Possible)	
Mean	27.1
Standard Deviation	10.7
Students Scoring 30 Points or More	42.2%
Students Scoring 36 Points or More	22.3%
Multiple Choice (30 Possible Points)	
Mean	14.7
Standard Deviation	5.7
Students Scoring 15 Points or More	49.7%
Students Scoring 18 Points or More	31.0%
Performance Based (30 Possible Points)	
Mean	12.3
Standard Deviation	6.5
Students Scoring 15 Points or More	36.6%
Students Scoring 18 Points or More	22.3%
Multiple Choice=a+b•(Performance Based)	
β	0.522
ε	0.440
t-statistic	30.191
p	(0.000)*

*P-value 0.000 designates p<0.001

TABLE 2
Overall Results
Calculus 3A
N=553

	β	ε	t-statistic
Final Grade=$\alpha+\beta$ x			
Final Recorded Score (x)	0.048	0.647	8.475**
Adjusted R^2	0.114		
Final Grade=$\alpha+\beta$ x			
Multiple Choice (x)	0.085	0.607	8.472**
Adjusted R^2	0.110		
Final Grade=$\alpha+\beta$ x			
Performanced Based (x)	0.052	0.330	5.662**
Adjusted R^2	0.053		
Final Grade=$\alpha+\beta_1 x_1+\beta_2 x_2$			
Multiple Choice (x_1)	0.073	0.521	6.759**
Performanced Based (x_2)	0.026	0.165	2.772**
Adjusted R^2 (p)	0.124		(0.000)***

*Significant at the 5% level
**Significant at the 1% level
***P-value 0.000 designates p<0.001

TABLE 3
Performanced Based
Calculus 3A
N=517
Final Grade=$\alpha+\beta \bullet x$

	β	ε	t-statistic
Algebra (x_1)	0.043	0.050	1.148
Function (x_2)	0.180	0.1218	5.434**
Proportion (x_3)	0.181	0.0934	2.301*
Geometry (x_4)	0.039	0.0143	0.405
Adjusted R^2 (p)	0.064		(0.000)***

*Significant at the 5% level
**Significant at the 1% level
***P-value 0.000 designates p<0.001

TABLE 4
Multiple Choice
Calculus 3A
N=527

Final Grade=$\alpha+\beta \bullet x$

	β	ε	t-statistic
Functions (x_1)	0.052	0.189	3.021**
Graphing			
9			
10			
21#			
22#			
23#			
27			
28			
29#			
Symbolic Properties			
4			
19			
30			
Trigonometry			
20			
22#			
26			
Logarithms			
5			
23#			
29#			
Numerical Data (x_2)	0.243	0.169	1.835
2#			
3#			
Symbolic Manipulations (x_3)	0.081	0.291	3.330**
1			
2#			
6			
7			
8			
11			
12			
13			
14			
17			
18			
21#			
24			
25#			
Word Problems (x_4)	-0.082	-0.067	-0.962
3#			
16#			
25#			
Geometry (x_5)	0.243	0.092	2.951**
15			
16#			
Adjusted R^2 (p)	0.114		(0.000)***

#Note that the questions that fall into more than one category have been included in all relevant categories for the regression. This may cause some degree of multicollinearity among the independent variables

*Significant at the 5% level
**Significant at the 1% level
***P-value 0.000 designates p<0.001

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TABLE 5
Multiple Choice
Calculus 3A
N=527
Final Grade=$\alpha+\beta \cdot x$

	β	ε	t-statistic
Functions			
Graphing (x_1)	0.030	0.057	0.694
9			
10			
21#			
22#			
23#			
27			
28			
29#			
Symbolic Properties (x_2)	0.188	0.126	2.805**
4			
19			
30			
Trigonometry (x_3)	0.085	0.048	1.429
20			
22#			
26			
Logarithms (x_4)	0.017	0.009	0.249
5			
23#			
29#			
Numerical Data (x_5)	0.237	0.165	1.792
2#			
3#			
Symbolic Manipulations (x_6)	0.077	0.277	3.156**
1			
2#			
6			
7			
8			
11			
12			
13			
14			
17			
18			
21#			
24			
25#			
Word Problems (x_7)	-0.090	-0.074	-1.054
3#			
16#			
25#			
Geometry (x_8)	0.221	0.084	2.651**
15			
16#			
Adjusted R^2 (p)	0.117		(0.000)***

#Note that the questions that fall into more than one category have been included in all relevant categories for the regression. This may cause some degree of multicollinearity among the independent variables

*Significant at the 5% level
**Significant at the 1% level
***P-value 0.000 designates p<0.001

TABLE 6
ADT and Pass/Fail Percentages of Calculus 3A Students
N=553

Recorded Final Score	Frequency of Failing Students	Cumulative Percent of Failing Students	Frequency of Passing Students	Cumulative Percent of Passing Students	Total	Total Cumulative Percent	Percent Scoring ≤ x who Failed
0	0	.0%	0	.0%	0	0.0%	
1	0	.0%	0	.0%	0	0.0%	
2	0	.0%	0	.0%	0	0.0%	
3	0	.0%	0	.0%	0	0.0%	
4	1	.8%	0	.0%	1	0.2%	100.0%
5	0	.8%	0	.0%	0	0.2%	100.0%
6	0	.8%	0	.0%	0	0.2%	100.0%
7	0	.8%	0	.0%	0	0.2%	100.0%
8	1	1.6%	1	.2%	2	0.5%	66.7%
9	0	1.6%	0	.2%	0	0.5%	66.7%
10	1	2.4%	1	.5%	2	0.9%	60.0%
11	1	3.2%	1	.7%	2	1.3%	57.1%
12	2	4.8%	1	.9%	3	1.8%	60.0%
13	2	6.4%	0	.9%	2	2.2%	66.7%
14	1	7.2%	1	1.2%	2	2.5%	64.3%
15	2	8.8%	0	1.2%	2	2.9%	68.8%
16	0	8.8%	0	1.2%	0	2.9%	68.8%
17	2	10.4%	1	1.4%	3	3.4%	68.4%
18	2	12.0%	3	2.1%	5	4.3%	62.5%
19	4	15.2%	3	2.8%	7	5.6%	61.3%
20	1	16.0%	0	2.8%	1	5.8%	62.5%
21	3	18.4%	3	3.5%	6	6.9%	60.5%
22	4	21.6%	6	4.9%	10	8.7%	56.3%
23	2	23.2%	2	5.4%	4	9.4%	55.8%
24	1	24.0%	6	6.8%	7	10.7%	50.8%
25	2	25.6%	0	6.8%	2	11.0%	52.5%
26	5	29.6%	9	8.9%	14	13.6%	49.3%
27	4	32.8%	3	9.6%	7	14.8%	50.0%
28	3	35.2%	7	11.2%	10	16.6%	47.8%
29	2	36.8%	21	16.1%	23	20.8%	40.0%
30	2	38.4%	8	18.0%	10	22.6%	38.4%

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TABLE 6
Continued

Recorded Final Score	Frequency of Failing Students	Cumulative Percent of Failing Students	Frequency of Passing Students	Cumulative Percent of Passing Students	Total	Total Cumulative Percent	Percent Scoring ≤ x who Failed
31	7	44.0%	20	22.7%	27	27.5%	36.2%
32	4	47.2%	22	27.8%	26	32.2%	33.1%
33	11	56.0%	21	32.7%	32	38.0%	33.3%
34	10	64.0%	28	39.3%	38	44.8%	32.3%
35	8	70.4%	27	45.6%	35	51.2%	31.1%
36	5	74.4%	22	50.7%	27	56.1%	30.0%
37	4	77.6%	23	56.1%	27	60.9%	28.8%
38	7	83.2%	23	61.4%	30	66.4%	28.3%
39	5	87.2%	20	66.1%	25	70.9%	27.8%
40	5	91.2%	17	70.1%	22	74.9%	27.5%
41	1	92.0%	23	75.5%	24	79.2%	26.3%
42	2	93.6%	12	78.3%	14	81.7%	25.9%
43	2	95.2%	18	82.5%	20	85.4%	25.2%
44	3	97.6%	9	84.6%	12	87.5%	25.2%
45	1	98.4%	12	87.4%	13	89.9%	24.7%
46	1	99.2%	8	89.3%	9	91.5%	24.5%
47	0	99.2%	10	91.6%	10	93.3%	24.0%
48	0	99.2%	8	93.5%	8	94.8%	23.7%
49	0	99.2%	3	94.2%	3	95.3%	23.5%
50	0	99.2%	5	95.3%	5	96.2%	23.3%
51	0	99.2%	8	97.2%	8	97.6%	23.0%
52	1	100.0%	6	98.6%	7	98.9%	22.9%
53	0	100.0%	3	99.3%	3	99.5%	22.7%
54	0	100.0%	0	99.3%	0	99.5%	22.7%
55	0	100.0%	1	99.5%	1	99.6%	22.7%
56	0	100.0%	2	100.0%	2	100.0%	22.6%
57	0	100.0%	0	100.0%	0	100.0%	22.6%
58	0	100.0%	0	100.0%	0	100.0%	22.6%
59	0	100.0%	0	100.0%	0	100.0%	22.6%
60	0	100.0%	0	100.0%	0	100.0%	22.6%

TABLE 7
Overall Results
Calculus 34A
N=175

	β	ε	t-statistic
Final Grade=$\alpha+\beta$ x			
Final Recorded Score (x)	0.040	0.412	4.455**
Adjusted R^2	0.098		
Final Grade=$\alpha+\beta$ x			
Multiple Choice (x)	0.056	0.306	3.604**
Adjusted R^2	0.064		
Final Grade=$\alpha+\beta$ x			
Performanced Based (x)	0.049	0.237	3.452**
Adjusted R^2	0.059		
Final Grade=$\alpha+\beta_1 x_1+\beta_2 x_2$			
Multiple Choice (x_1)	0.044	0.240	2.732**
Performanced Based (x_2)	0.037	0.179	2.535*
Adjusted R^2 (p)	0.093		(0.000)***

*Significant at the 5% level
**Significant at the 1% level
***P-value 0.000 designates p<0.001

TABLE 8
Performanced Based
Calculus 34A
N=175
Final Grade=$\alpha+\beta \bullet \mathbf{x}$

	β	ε	t-statistic
Algebra (x_1)	0.087	0.071	1.894
Function (x_2)	0.121	0.045	2.071*
Proportion (x_3)	0.171	0.068	1.578
Geometry (x_4)	0.079	0.020	0.621
Adjusted R^2 (p)	0.046		(0.018)

*Significant at the 5% level

TABLE 9
Multiple Choice
Calculus 34A
N=169
Final Grade=$\alpha+\beta \bullet x$

	β	ε	t-statistic
Functions (x_1)	0.028	0.075	1.109
Graphing			
9			
10			
21#			
22#			
23#			
27			
28			
29#			
Symbolic Properties			
4			
19			
30			
Trigonometry			
20			
22#			
26			
Logarithms			
5			
23#			
29#			
Numerical Data (x_2)	-0.278	-0.169	-1.642
2#			
3#			
Symbolic Manipulations (x_3)	0.105	0.289	3.324**
1			
2#			
6			
7			
8			
11			
12			
13			
14			
17			
18			
21#			
24			
25#			
Word Problems (x_4)	-0.059	-0.040	-1.373
3#			
16#			
25#			
Geometry (x_5)	0.258	0.070	1.895
15			
16#			
Adjusted R^2 (p)	0.087		(0.001)

#Note that the questions that fall into more than one category have been included in all relevant categories for the regression. This may cause some degree of multicollinearity among the independent variables

*Significant at the 5% level
**Significant at the 1% level
***P-value 0.000 designates p<0.001

TABLE 10
Multiple Choice
Calculus 34A
N=169
Final Grade=$\alpha+\beta *x$

	β	ε	t-statistic
Functions			
Graphing (x_1)	0.006	0.009	1.108
9			
10			
21#			
22#			
23#			
27			
28			
29#			
Symbolic Properties (x_2)	-0.080	-0.043	-0.844
4			
19			
30			
Trigonometry (x_3)	0.070	0.028	0.822
20			
22#			
26			
Logarithms (x_4)	0.106	0.035	1.054
5			
23#			
29#			
Numerical Data (x_5)	-0.277	-0.168	-1.612
2#			
3#			
Symbolic Manipulations (x_6)	0.107	0.294	3.364**
1			
2#			
6			
7			
8			
11			
12			
13			
14			
17			
18			
21#			
24			
25#			
Word Problems (x_7)	-0.146	-0.100	-1.252
3#			
16#			
25#			
Geometry (x_8)	0.254	0.069	1.858
15			
16#			
Adjusted R^2 (ρ)	0.081		(0.005)

#Note that the questions that fall into more than one category have been included in all relevant categories for the regression. This may cause some degree of multicollinearity among the independent variables

*Significant at the 5% level
**Significant at the 1% level
***P-value 0.000 designates p<0.001

TABLE 11
Overall Results
Mathematics 15
N=256

	β	ε	t-statistic
Final Grade=$\alpha+\beta$ x			
Final Recorded Score (x)	0.041	0.344	4.907**
Adjusted R^2	0.083		
Final Grade=$\alpha+\beta$ x			
Multiple Choice (x)	0.058	0.271	4.085**
Adjusted R^2	0.058		
Final Grade=$\alpha+\beta$ x			
Performanced Based (x)	0.041	0.153	3.428**
Adjusted R^2	0.041		
Final Grade=$\alpha+\beta_1 x_1+\beta_2 x_2$			
Multiple Choice (x_1)	0.051	0.238	3.537**
Performanced Based (x_2)	0.033	0.123	2.769**
Adjusted R^2 (p)	0.082		(0.000)***

*Significant at the 5% level
**Significant at the 1% level
***P-value 0.000 designates p<0.001

TABLE 12
Performanced Based
Mathematics 15
N=256
Final Grade=$\alpha+\beta \bullet x$

	β	ε	t-statistic
Algebra (x_1)	0.069	0.0451	1.871
Function (x_2)	0.014	0.0043	0.253
Proportion (x_3)	0.222	0.088	2.627**
Geometry (x_4)	0.215	0.0442	1.940
Adjusted R^2 (p)	0.042		(0.005)

**Significant at the 1% level

TABLE 13
Multiple Choice
Mathematics 15
N=276
Final Grade=$\alpha+\beta \bullet x$

	β	ε	t-statistic
Functions (x_1)	0.030	0.062	1.386
Graphing			
9			
10			
21#			
22#			
23#			
27			
28			
29#			
Symbolic Properties			
4			
19			
30			
Trigonometry			
20			
22#			
26			
Logarithms			
5			
23#			
29#			
Numerical Data (x_2)	0.061	0.039	0.582
2#			
3#			
Symbolic Manipulations (x_3)	0.088	0.200	3.220**
1			
2#			
6			
7			
8			
11			
12			
13			
14			
17			
18			
21#			
24			
25#			
Word Problems (x_4)	-0.037	-0.023	-0.352
3#			
16#			
25#			
Geometry (x_5)	-0.099	-0.024	-0.975
15			
16#			
Adjusted R^2 (p)	0.067		(0.000)***

#Note that the questions that fall into more than one category have been included in all relevant categories for the regression. This may cause some degree of multicollinearity among the independent variables

*Significant at the 5% level
**Significant at the 1% level
***P-value 0.000 designates p<0.001

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TABLE 14
Multiple Choice
Mathematics 15
N=276
Final Grade=$\alpha+\beta*x$

	β	ε	t-statistic
Functions			
Graphing (x_1)	0.033	0.038	0.702
9			
10			
21#			
22#			
23#			
27			
28			
29#			
Symbolic Properties (x_2)	0.010	0.004	1.427
4			
19			
30			
Trigonometry (x_3)	-0.118	-0.034	-1.597
20			
22#			
26			
Logarithms (x_4)	0.131	0.026	1.167
5			
23#			
29#			
Numerical Data (x_5)	0.056	0.036	0.538
2#			
3#			
Symbolic Manipulations (x_6)	0.084	0.191	3.076**
1			
2#			
6			
7			
8			
11			
12			
13			
14			
17			
18			
21#			
24			
25#			
Word Problems (x_7)	-0.031	-0.020	-0.299
3#			
16#			
25#			
Geometry (x_8)	-0.107	-0.026	-1.068
15			
16#			
Adjusted R^2 (p)	0.081		(0.000)***

#Note that the questions that fall into more than one category have been included in all relevant categories for the regression. This may cause some degree of multicollinearity among the independent variables

*Significant at the 5% level
**Significant at the 1% level
***P-value 0.000 designates p<0.001

TABLE 15
Overall Results
Chemistry 1A
N=594

	β	ε	t-statistic
Final Grade=$\alpha+\beta$ x			
Final Recorded Score (x)	0.031	0.375	10.199**
Adjusted R^2	0.148		
Final Grade=$\alpha+\beta$ x			
Multiple Choice (x)	0.061	0.398	10.590**
Adjusted R^2	0.158		
Final Grade=$\alpha+\beta$ x			
Performanced Based (x)	0.039	0.218	7.324**
Adjusted R^2	0.082		
Final Grade=$\alpha+\beta_1 x_1+\beta_2 x_2$			
Multiple Choice (x_1)	0.052	0.339	7.672**
Performanced Based (x_2)	0.013	0.073	2.202*
Adjusted R^2 (p)	0.163		(0.000)***

*Significant at the 5% level
**Significant at the 1% level
***P-value 0.000 designates p<0.001

TABLE 16
Performanced Based
Chemistry 1A
N=594
Final Grade=$\alpha+\beta \bullet \mathbf{x}$

	β	ε	t-statistic
Algebra (x_1)	0.066	0.0676	2.901**
Function (x_2)	0.106	0.0593	4.439**
Proportion (x_3)	0.080	0.0384	1.693
Geometry (x_4)	0.122	0.040	2.020*
Adjusted R^2 (p)	0.080		(0.000)***

*Significant at the 5% level
**Significant at the 1% level
***P-value 0.000 designates p<0.001

TABLE 17
Multiple Choice
Chemistry 1A
N=578
Final Grade=$\alpha+\beta$ *x

	β	ε	t-statistic
Functions (x_1)	0.032	0.102	2.934**
Graphing			
9			
10			
21#			
22#			
23#			
27			
28			
29#			
Symbolic Properties			
4			
19			
30			
Trigonometry			
20			
22#			
26			
Logarithms			
5			
23#			
29#			
Numerical Data (x_2)	0.060	0.043	0.834
2#			
3#			
Symbolic Manipulations (x_3)	0.066	0.216	4.444**
1			
2#			
6			
7			
8			
11			
12			
13			
14			
17			
18			
21#			
24			
25#			
Word Problems (x_4)	-0.012	-0.010	-0.220
3#			
16#			
25#			
Geometry (x_5)	0.150	0.050	2.674**
15			
16#			
Adjusted R^2 (p)	0.161		(0.000)***

#Note that the questions that fall into more than one category have been included in all relevant categories for the regression. This may cause some degree of multicollinearity among the independent variables

*Significant at the 5% level
**Significant at the 1% level
***P-value 0.000 designates p<0.001

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TABLE 18
Multiple Choice
Chemistry 1A
N=578

Final Grade=$\alpha+\beta \cdot x$

	β	ε	t-statistic
Functions			
Graphing (x_1)	0.169	0.287	-0.625
9			
10			
21#			
22#			
23#			
27			
28			
29#			
Symbolic Properties (x_2)	0.048	0.029	1.126
4			
19			
30			
Trigonometry (x_3)	0.135	0.066	3.362**
20			
22#			
26			
Logarithms (x_4)	0.051	0.020	1.129
5			
23#			
29#			
Numerical Data (x_5)	0.060	0.043	0.827
2#			
3#			
Symbolic Manipulations (x_6)	0.067	0.220	4.558**
1			
2#			
6			
7			
8			
11			
12			
13			
14			
17			
18			
21#			
24			
25#			
Word Problems (x_7)	-0.016	-0.013	-0.290
3#			
16#			
25#			
Geometry (x_8)	0.142	0.047	2.537*
15			
16#			
Adjusted R^2 (p)	0.169		(0.000)***

#Note that the questions that fall into more than one category have been included in all relevant categories for the regression. This may cause some degree of multicollinearity among the independent variables

*Significant at the 5% level
**Significant at the 1% level
***P-value 0.000 designates p<0.001

Appendix 6

The Precalculus Diagnostic Test
as Published by
CSU/UC Mathematics Diagnostic Testing Project

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CSU/UC MATHEMATICS DIAGNOSTIC TESTING PROJECT

PRECALCULUS DIAGNOSTIC TEST

A READINESS TEST FOR CALCULUS
CALCULATOR ALLOWED BUT NOT REQUIRED

PC60 – A
1993

A suggested time for this test is approximately 90 minutes.

INSTRUCTIONS

1. Wait until you are told to start before beginning the test.
2. Work each problem and then on the answer sheet mark the space which corresponds to your answer. The test booklet, the answer sheet, and all scratch paper must be turned in when the test is finished. DO NOT WRITE IN THIS BOOKLET.
3. For each problem you are to select the best response from the given choices.
4. If you find certain questions very time consuming, leave them temporarily. Come back to them after you have gone through the entire test if you have time.
5. In scoring the test, only correct answers will be counted. If you have no idea which of the answers to a given question is the correct one, you should leave the question blank.

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PC60A93 Test Type 0116093 Precalculus – 60 Questions

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PRECALCULUS DIAGNOSTIC TEST - 60 QUESTIONS - 90 MINUTES

1. If $a = -3$, then $|a - 1| - |3 - 2a| =$

(A) -5 (B) -1 (C) 1 (D) 5 (E) 13

2. $\left(\frac{26}{75x^4}\right)\left(\frac{3x^3}{52y}\right)(25y^2) =$

(A) $\frac{3x}{2y}$ (B) $\frac{y}{2x}$ (C) $\frac{3y}{2x}$ (D) $\frac{y^2}{2x}$ (E) $\frac{y}{6x}$

3. How many seconds does it take an object to travel 100 centimeters at an average speed of 20 centimeters per second?

(A) 0.2 (B) 5 (C) 50 (D) 80 (E) 2,000

4. If $f(x) = 3x - 4$ and $g(x) = x^2 + 1$, then $f(g(2)) =$

(A) 2 (B) 5 (C) 7 (D) 10 (E) 11

5. If $a = 4$ and $b = -3$, then $\frac{a^2 - ab}{b^2 - ab} =$

(A) $-\frac{28}{3}$ (B) $-\frac{4}{3}$ (C) $\frac{3}{4}$ (D) $\frac{4}{3}$ (E) $\frac{16}{9}$

6. If $\frac{3}{2}(x + 2) - 2 = 7x$, then $x =$

(A) $-\frac{2}{11}$ (B) 0 (C) $\frac{2}{11}$ (D) $\frac{4}{11}$ (E) $\frac{1}{2}$

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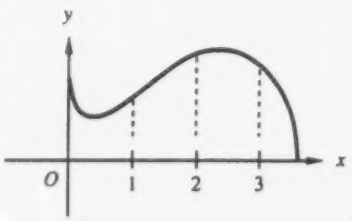
7. In the system of equations $\begin{cases} x - 3y = -1 \\ 2x + y = 12 \end{cases}$, $y =$

(A) -5 (B) -2 (C) 0 (D) 2 (E) 5

8. In a room with 280 people there are $1\frac{1}{3}$ times as many men as women. How many men are in the room?

(A) 40 (B) 70 (C) 160 (D) 210 (E) 240

9. The graph of the function $y = f(x)$ is shown in the figure to the right. Which of the following is true?



(A) $f(1) = f(3)$ (B) $f(2) < f(1)$

(C) $f(2) < f(3)$ (D) $f(3) < f(1)$

(E) $f(3) < f(2)$

10. $\left(\frac{x+3}{x-3}\right)\left(\frac{x^2-9}{x^2+2x-3}\right) =$

(A) $-\frac{3}{2x}$ (B) $\frac{x+3}{x-1}$ (C) $\frac{x-3}{x-1}$ (D) $\frac{x+3}{x+1}$ (E) $\frac{x^2+2x-3}{(x-3)^2}$

11. The inequality $3 - 2x < 5$ is equivalent to

(A) $x > -1$ (B) $x > -4$ (C) $x < 0$ (D) $x < -1$ (E) $x < -4$

12. $(8x^6y^{-3})^{1/3} =$

(A) $\frac{8x^2}{y}$ (B) $\frac{2x^2}{y}$ (C) $\frac{x^2}{2y}$ (D) $\frac{y}{2x^2}$ (E) $2x^2y$

GO ON TO THE NEXT PAGE.

13. The inequality $x^2 > 4$ is equivalent to

(A) $x < -2$ (B) $x > 2$ (C) $-2 < x < 2$

(D) $x < 0$ or $x > 2$ (E) $x < -2$ or $x > 2$

14. When $2x^2 - 7x + 4$ is divided by $x - 3$ the remainder is

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

15. The inequality $|x - 3| < 4$ is equivalent to

(A) $-7 < x < 7$ (B) $-7 < x < 1$ (C) $-1 < x < 7$

(D) $2 < x < 4$ (E) $3 < x < 4$

16. $\dfrac{x^{2a+3}}{x^{a-3}} =$

(A) x^6 (B) x^a (C) x^{3a} (D) x^{a-6} (E) x^{a+6}

17. $\dfrac{2w-1}{w+1} - \dfrac{1}{w-1} =$

(A) $\dfrac{2w(w-2)}{w^2-2w+1}$ (B) $\dfrac{2w(w-2)}{w^2-1}$ (C) $\dfrac{2w}{w^2-1}$

(D) $\dfrac{2(w-1)}{w+1}$ (E) $\dfrac{2}{w+1}$

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18. If $b > 0$, then $\sqrt[4]{\frac{1}{\sqrt[3]{b^{18}}}} =$

(A) $\frac{1}{b^{11}}$ (B) $\frac{1}{b}$ (C) $\frac{1}{b\sqrt{b}}$ (D) $-\frac{1}{b}$ (E) $-b\sqrt{b}$

19. If $3 = 10^t$, then $t =$

(A) $\frac{10}{\sqrt{3}}$ (B) $\frac{3}{\sqrt{10}}$ (C) 10^3 (D) $\log_{10} 3$ (E) $\log_{10} \frac{1}{3}$

20. $\left(\frac{2x^0y^{-2}}{z}\right)^{-3} =$

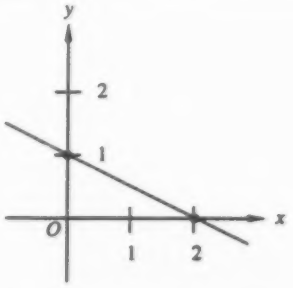
(A) 0 (B) $\frac{-8}{y^2z^3}$ (C) $\frac{z^3}{8x^3y^5}$ (D) $\frac{y^6z^3}{8}$ (E) $\frac{8y^6}{z^3}$

21. An equation of the line in the figure shown to the right is

(A) $\frac{1}{2}x + y = 1$ (B) $x + \frac{1}{2}y = 1$

(C) $x + y = 1$ (D) $x + 2y = 1$

(E) $2x + y = 1$



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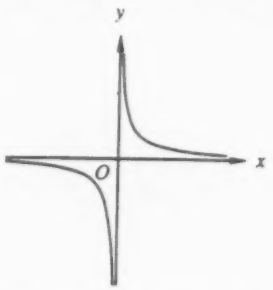
22. Which of the following could be an equation for the graph shown in the figure to the right?



(A) $y = \frac{2}{x}$ (B) $y = -\frac{2}{x}$

(C) $y = \frac{2}{x^2}$ (D) $y = 2^{-x}$

(E) $y = -\log_2 x$

23. If $f(x) = \frac{6}{x-1}$, for what value of x does $f(x) = 5$?

(A) $\frac{1}{5}$ (B) $\frac{5}{11}$ (C) $\frac{7}{5}$ (D) $\frac{3}{2}$ (E) $\frac{11}{5}$

24. $x^{-1}(x^{-2} + x^{-3}) =$

(A) $\frac{1}{x^6}$ (B) $\frac{1}{x^7}$ (C) $\frac{1}{x^3 + x^4}$ (D) $\frac{1}{x^3} + \frac{1}{x^4}$ (E) $x^2 + x^3$

25. A rectangular field with area 300 square meters has a length that is five meters more than its width. If the width of the field is w meters, then an equation that can be used to determine the value of w is

(A) $w^2 - 1500 = 0$ (B) $w^2 - 295 = 0$ (C) $w^2 + 5w - 300 = 0$

(D) $w^2 - 5w - 300 = 0$ (E) $5w^2 - 300 = 0$

26. What number must be added to $x^2 + 3x$ to form a perfect square?

(A) $\frac{3}{2}$ (B) $\frac{9}{4}$ (C) $\frac{9}{2}$ (D) 6 (E) 9

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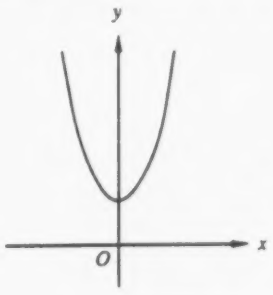
27. Which of the following could be an equation for the graph shown in the figure to the right?



(A) $y = x^2 + 3$ (B) $y = x^2 - 3$

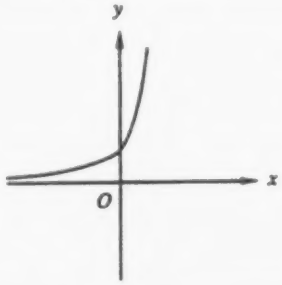
(C) $y = (x - 3)^2$ (D) $y = 3 - x^2$

(E) $x = y^2 + 3$

28. $$\frac{\dfrac{15 - 3x}{x + 5}}{\dfrac{5x - x^2}{5x + 1}} =$$

(A) $\frac{3}{x}$ (B) $\frac{3(5x + 1)}{x + 5}$ (C) $\frac{3x(5 - x)}{5x + 1}$ (D) $\frac{3(5x + 1)}{x(x + 5)}$ (E) $\frac{3x(5 - x)^2}{(x + 5)(5x + 1)}$

29. Which of the following could be an equation for the graph shown in the figure to the right?



(A) $y = x^2$ (B) $y = 2^x - 1$

(C) $y = 2^x$ (D) $y = 2^{-x}$

(E) $y = \log_2 x$

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30. If $x - \frac{bx}{c} = a$ and $b \neq c$, then $x =$

(A) $\frac{ac}{c-b}$ (B) $\frac{a}{c-b}$ (C) $\frac{ac}{b}$ (D) $a - \frac{b}{c}$ (E) $\frac{ac}{b} - 1$

31. For *how many* different values of θ between 0 and 2π radians is $\sin\theta = \cos\theta$?

(A) none (B) one (C) two (D) three (E) four

32. The perimeter of a square is directly proportional to the length of a side and its area is directly proportional to the square of the length of a side. If the perimeter of a square is multiplied by 3, then its area is multiplied by

(A) $\frac{1}{9}$ (B) $\frac{1}{3}$ (C) 3 (D) 6 (E) 9

33. If $\frac{1}{2}$ is $\frac{3}{4}$ of $\frac{5}{6}$ of a certain number, then that number is

(A) $\frac{5}{4}$ (B) $\frac{4}{5}$ (C) $\frac{3}{4}$ (D) $\frac{3}{5}$ (E) $\frac{5}{16}$

34. If $8^{2x+1} = 4^{1-x}$, then $x =$

(A) $-\frac{1}{8}$ (B) $-\frac{1}{5}$ (C) 0 (D) $\frac{1}{5}$ (E) $\frac{1}{8}$

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35. If a and b are in the domain of a function f and $f(a) < f(b)$, which of the following must be true?

(A) $a = 0$ or $b = 0$ (B) $a < b$ (C) $a \leq b$ (D) $a \neq b$ (E) $a = b$

36. One of the roots of $x^2 + x - 3 = 0$ is

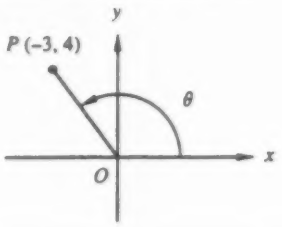
(A) $\frac{1+\sqrt{13}}{2}$ (B) $\frac{1-2\sqrt{3}}{2}$ (C) $\frac{-1+\sqrt{3}}{2}$

(D) $\frac{-1-2\sqrt{3}}{2}$ (E) $\frac{-1-\sqrt{13}}{2}$

37. In the figure shown to the right, $\tan\theta =$

(A) $-\frac{4}{3}$ (B) $-\frac{3}{4}$ (C) $-\frac{3}{5}$

(D) $\frac{3}{4}$ (E) $\frac{4}{3}$



38. If $x \neq 2$ and $(x-2)^2(x+1) + 3(x-2)(x+1) = (x-2)P$, then $P =$

(A) $(x-2)(x+1)$ (B) $x^2 - 1$ (C) $(x+1)^2$ (D) $x^2 + 1$ (E) $x + 2$

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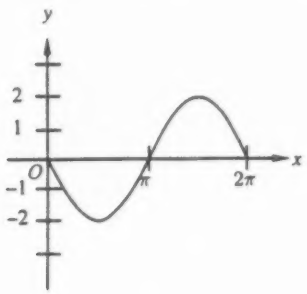
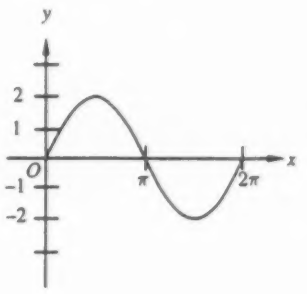
39. Which of the following could be a portion of the graph of $y = -2\sin x$?

(A)



(B)



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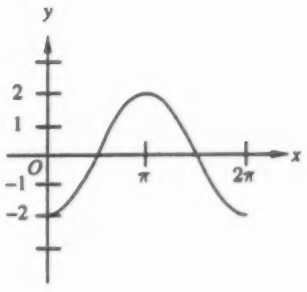
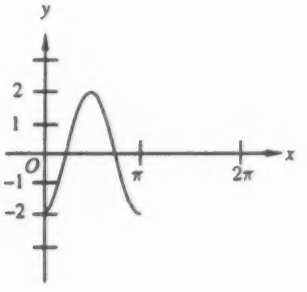
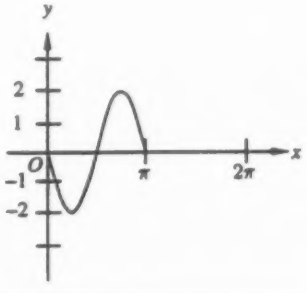


(D)



(E)



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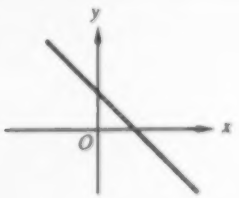
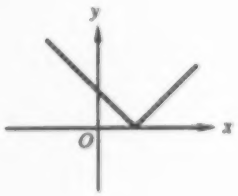
40. The graph of $y = f(x)$ is shown in the figure to the right. Which of the following is the graph of $y = |f(x)|$?

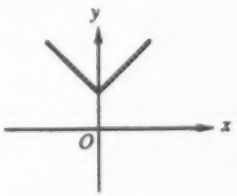


(A)

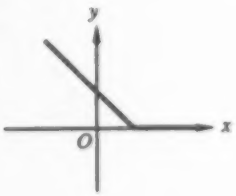
(B)



(C)



(D)



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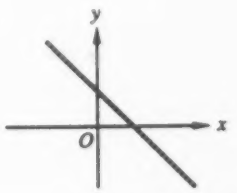

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41. $\log_2 64 =$

(A) $\frac{1}{6}$ (B) 6 (C) 8 (D) 32 (E) 128

42. If $\log w = \log x + \frac{1}{2}\log y$, then $w =$

(A) $\frac{xy}{2}$ (B) $(xy)^{1/2}$ (C) $(x+y)^{1/2}$ (D) $xy^{1/2}$ (E) $x + \frac{1}{2}y$

43. One of the roots of $2x^2 + 3x - 2 = 0$ is

(A) -1 (B) $-\frac{1}{2}$ (C) $\frac{1}{2}$ (D) 1 (E) 2

44. If $f(x) = x^2 - x + 1$, then $f(2a) =$

(A) $2a^2 - 2a + 1$ (B) $2a^2 - 2a + 2$ (C) $4a^2 - 2a + 1$

(D) $4a^2 - 2a - 1$ (E) $4a^2 + 2a + 1$

45. $\sin(\theta + 2\pi) =$

(A) $-\cos\theta$ (B) $-\sin\theta$ (C) $\sin 2\theta$ (D) $\cos\theta$ (E) $\sin\theta$

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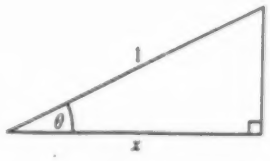
46. In the figure shown to the right, $\sin\theta =$

(A) $\sqrt{1+x^2}$ (B) $\frac{\sqrt{1+x^2}}{x}$ (C) x

(D) $\sqrt{1-x^2}$ (E) $\frac{\sqrt{1-x^2}}{x}$



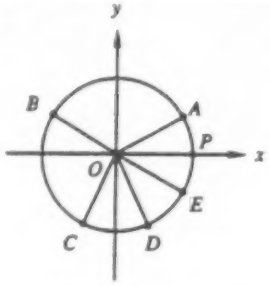
47. $\frac{y}{\sqrt[3]{5}} =$

(A) $\frac{y\sqrt[3]{25}}{5}$ (B) $\frac{y\sqrt[3]{25}}{25}$ (C) $\frac{y\sqrt[3]{5}}{5}$ (D) $\frac{\sqrt[3]{5y}}{5}$ (E) $\frac{y\sqrt{5}}{5}$

48. In the figure shown to the right, if $\cos\theta = -\frac{5}{6}$, which angle could best represent θ?

(A) $\angle POA$ (B) $\angle POB$ (C) $\angle POC$

(D) $\angle POD$ (E) $\angle POE$



49. $\sqrt{16x^6 + 4a^2x^4} =$

(A) $4x^3 + 2ax^2$ (B) $4x^4 + 2ax^2$ (C) $2x^2\sqrt{4x^2 + a^2}$

(D) $2ax^2\sqrt{4x^2 + 1}$ (E) $4x^2\sqrt{4x^2 + a^2}$

50. $\frac{\frac{d-c}{c}}{\frac{d}{d} - \frac{d}{c}} =$

(A) $\frac{-1}{cd(c+d)}$ (B) $\frac{1}{cd(c+d)}$ (C) $\frac{-cd}{c-d}$ (D) $\frac{cd}{c-d}$ (E) $\frac{-cd}{c+d}$

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51. $\dfrac{x-1}{x^2-2x}+\dfrac{1}{2x-4}=$

(A) $\dfrac{1}{2x(x-2)}$ (B) $\dfrac{x}{x^2-4}$ (C) $\dfrac{3x-2}{(x-2)^2}$ (D) $\dfrac{3x-2}{2x(x-2)}$ (E) $\dfrac{3x-2}{2x(x-2)^2}$

52. The graphs of the two equations $3x+y=2$ and $3x-9y=5$ are

(A) the same line (B) two perpendicular lines (C) two distinct parallel lines

(D) two intersecting lines which are not perpendicular (E) not straight lines

53. The perimeter of a rectangular field is 1,200 feet and the field is 50 feet longer than it is wide. What is the length of the field in feet?

(A) 250 (B) 275 (C) 300 (D) 325 (E) 625

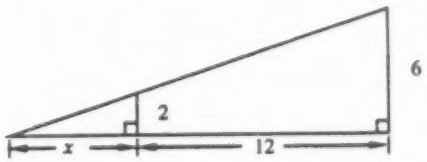
54. If $f(x)=x+3$ and $f(a+1)=2f(a)$, then $a=$

(A) -2 (B) 0 (C) 1 (D) 2 (E) 3

55. In the figure shown to the right, $x=$

(A) 2 (B) 4 (C) 6

(D) 9 (E) 10



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56. From September 1981 to September 1984 the enrollment at a particular school declined by 20 percent. If the number of students enrolled at that school in September 1984 was 720, what was the enrollment in September 1981?

(A) 144 (B) 576 (C) 700 (D) 864 (E) 900

57. In which of the following intervals is $\sin x > \cos x$ for every value of x?

(A) $\left(0, \frac{\pi}{2}\right)$ (B) $(0, \pi)$ (C) $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ (D) $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ (E) $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

58. If $\log_6 x + \log_6(x+2) = \log_6 24$, then $x =$

(A) 11 (B) 6 (C) 4 (D) -4 (E) -6

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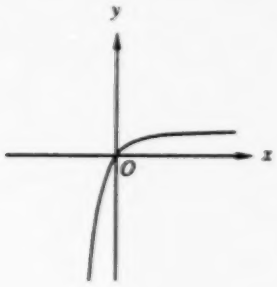
59. Which of the following could be a portion of the graph of $y = \log_e x$?

(A)



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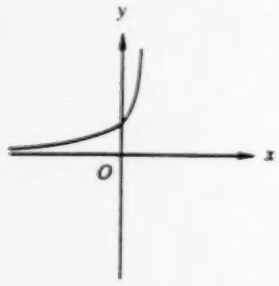

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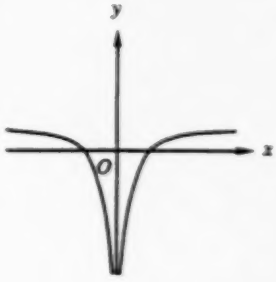

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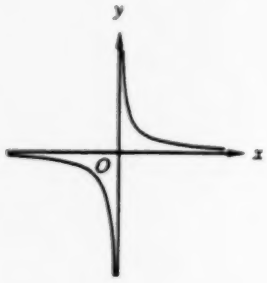
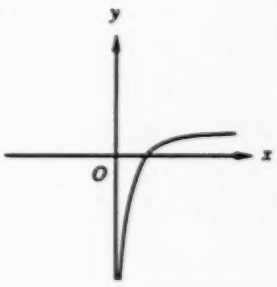


(E)



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60. $\sqrt{b}\ \sqrt[3]{b} =$

(A) b (B) $\sqrt[3]{b}$ (C) $\sqrt[6]{b}$ (D) $\sqrt[5]{b^3}$ (E) $\sqrt[6]{b^5}$

END OF EXAMINATION